Modelling of Low Frequency Muffler based on Acoustic Black Hole Effect

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To the memory of my beloved grandfather ***Kedarnath Sharma***

His words shall forever resonate in my mind;

"Choose your Path and Master it"

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Abstract

This thesis investigates the influence of acoustic metamaterials, or linings with graded properties, on the performance of open termination ducts and mufflers working within low frequency range. The functioning of such linings has been associated to the Acoustic Black Hole effect. These structures work on the principle of impedance matching, and in theory, have been realized to achieve total absorption of sound in closed terminating channels. This theoretical understanding, backed up with numerical analyses, has been reviewed to develop the concepts underlying this research work.

Firstly, a semi analytical model has been formulated for an open expansion chamber, embodied by the muffling section. Plane wave radiation incident through inlet end of the muffler, is made to undergo impedance matching while traversing through the metamaterial lined flare of varying wall admittance. The impact of lining, in absence and presence of losses due to visco-inertial and thermal energy exchanges has been analyzed. Parametric variations of flare shape and muffler dimensions have been compared. Finally, considering particular cases, comparative analysis with finite element method based model has been made. The semi-analytical models have been validated against numerical results for acoustical properties such as transmission loss and reflection and absorption coefficients in the transmission regime, while observing a good agreement between them.

It has been concluded that the structured lined flare has higher performance capability than the simple expansion chamber. Power law function based flare shape can be considered to be steady in operation over the low frequency range. The findings of this thesis show that the application of acoustic metamaterials for sound absorbing or muffling devices has a promising future. Thus, industrial ducts, vents and mufflers that form a major part of such systems, when augmented with the metamaterials can hopefully yield quieter machineries in times to come.

Chapter 1

Introduction

1.1. Motivation

To sustain the regulatory compliance norms, continual efforts are being taken to install and maintain quieter systems and machinery. As identified, when it comes to sound attenuating devices, issues (Leventhall, 2003) in the low frequency range have to be dealt with. Due to their nature to travel for long distances and the diminutive fluctuations, make their detection problematic and absorption difficult. The low frequency waves have also been reported to adversely impact human (Berglund, Hassmén and Job, 1996) and animal health (Francis and Barber, 2013) alike.

Besides automotive (Bentleypublishers.com, 2005) and aviation (Pawsey, 2016), other sectors where components of low frequency noise are considered disturbing, include, industrial plants, HVAC systems, high-speed trains, wind turbines, blowers and compressors, ducts and vents and the likes. In several cases, the industrial mufflers and silencers have been not only the target areas but also many a times the go-to solutions for amelioration. On one hand, a muffler can be taken as an expensive noise quelling component, generally a burden to the driving system, while, on the other, its acceptance invites functionality concerns at low frequency whilst attempting to meet the stringent customer laid specifications. To achieve low overall noise levels, machine designers, fabricators and operators have developed hybrid mufflers, unlike the traditional ones; making use of best of both the absorptive and the reactive design elements (Panigrahi and Munjal, 2005).

The presently used absorbers, expansion chambers, mufflers, catalytic convertors or any such synonymously termed silencing systems, have structured designs and internal components. They have well established techniques to model, analyze, rebuild, repair and even retrofit in many instances of failure. However, in addition to keeping up with the dimensional constraints, managing the attenuation of low frequency noise at source and/or receiver ends, in combination with manipulating its flow along the path, the technological advancement for modern-day mufflers seems both challenging and indispensable. It is with this understanding that the quest for developing efficacious low frequency muffler designs applying advanced techniques, frames the motivation of this project.

1.2. Need Statement

The currently available expansion chambers or mufflers are well designed, analyzed using established techniques and demonstrate optimum performance. However, in light of growing stringency on the compliance norms and customer requirements, concerns have been raised for low frequency sound absorption, with particular focus on parameters such as system dimension and the overall expenses of the installation.

Hence the present study was designed to explore the recent technologies that can have a significant contribution to come up with novel techniques of attenuating sound within muffling systems. In particular, it was intended to investigate the impact of:

1. Acoustic metamaterials or linings with graded properties, as an improvised technique of attenuating sound
2. Dimensional and shape-based variations of simple muffling devices on the absorption of low frequency sound.

1.3. Objectives

Keeping the needs in mind, the main objectives were laid down for undertaking the present research work. These are as follows;

1. To review the currently under-research and in-practice technologies for sound absorption and noise control, with regards to mufflers and silencers.
2. To learn and understand the use of pertinent analytical tools and techniques essential to tackle engineering problems based on internally muffling, open tube terminating structures.
3. To study the design requirements and characteristic parameters of simple muffling devices, specifically in the low frequency domain.
4. To study the influence of acoustic metamaterial based lining on the acoustic properties of low frequency mufflers.
5. To conduct a comparative study and analyze the impact of metamaterial linings in combination with design parameters on sound absorption through mufflers.

1.4. Work Plan

A general methodology that was adopted to achieve the objectives of this project was a combination of several approaches. This was carried out not only to understand the most desirable techniques but also get acquainted with the best practices for modelling and analyses.

Initially, a basic understanding was established in terms of the theoretical basis of the muffler technology and the associated acoustics. This was followed by the conceptual study of the sound absorbing techniques using acoustic metamaterials. With this as the framework of the entire research work, the problem statement was approached as shown (See Figure 1.1).

Muffler technology was studied in terms of the types, shapes and design requirements particularly for the low frequency attenuation. The basis of sound absorbing properties

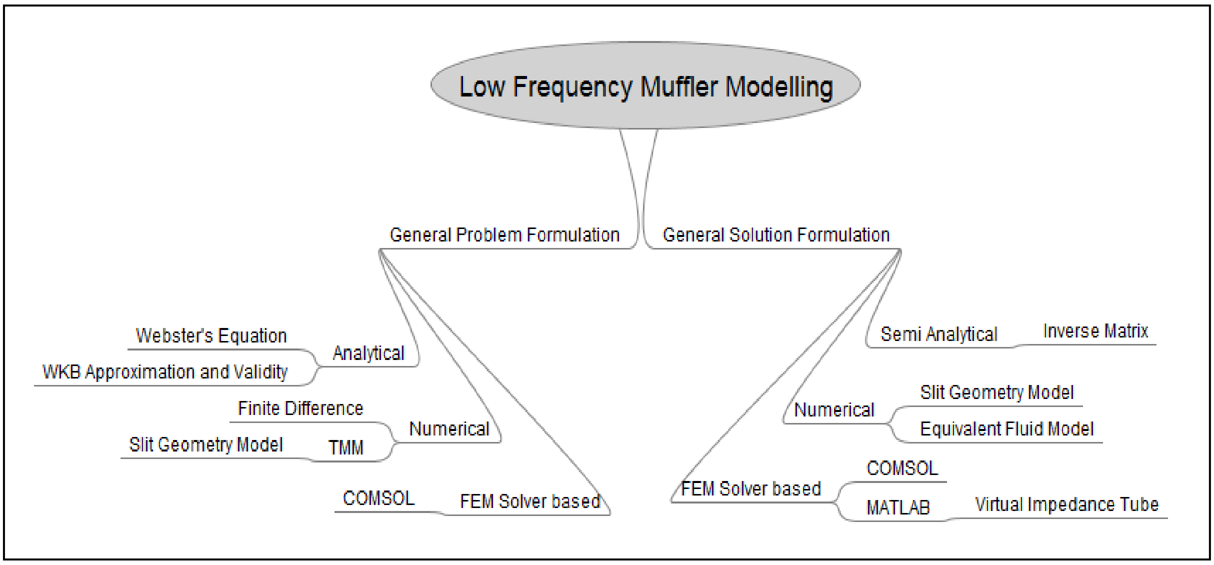

Figure 1.1: Framework of the research work

was reviewed through popular papers. Also the presently available sound absorbing techniques and the ones investigated so far were reviewed with regards to the acoustic metamaterial based lining or the so called, graded sub-structures.

The problem as defined for the current study have been considered as a two-part three-step process which includes the analytical/semi analytical method, the TMM based numerical method and the FEM solver based numerical method. These three approaches, as depicted in the flow diagram, have been used for the formulation of the general problem statement and then used to obtain the general and particular solution. An introduction to the experimental method has been described which will form a part of the future study.

The comparative analyses have been carried out to account for the several sound absorption parameters such as the reflection and absorption coefficients, the transmission loss, and the phase and SPL variations for the noise reduction. Essentially, these comparisons have been made for the dimensional parameter variations as also the shape of the muffling sections. Also the different approaches so considered have been compared to understand the effectiveness of each method. These methods have been appropriately considered for the absence and presence of losses due to medium, wherever applicable.

1.5. Thesis Outline

The present research work on the modelling of a low frequency muffler based on the Acoustic Black Hole effect is mainly a combination of theoretical, semi analytical and numerical models.

Chapter 1 (Introduction) gives the motivation behind the research carried out, defines the need statement and lays down the objectives. Based on the objectives set, this chapter briefs the plan of action enlisting the methodology to be followed all through the Masters project and last but not the least presents the outline of this thesis.

Chapter 2 (Review of Literature) reviews several concepts and approaches that have been referred to and have been utilized throughout the study. In the first sub-section, the fundamentals of sound attenuation, muffling and silencing which is the main focus of this project has been discussed. The second sub-section reviews in depth the history and present status of muffling and silencing devices. The various mechanisms of sound attenuation mainly associated to muffling and silencing have been mentioned under third sub-section. In the final sub-section, the approaches that have been followed through the course of study have been reviewed to present an idea of their applicability and formulation.

The following three chapters introduce the approach, details on how the model has been created, including the assumptions, approximations and particular considerations, the formulation of the problem, the application of the technique to derive the solution and a summary for each method of modelling specifying the key advantages and limitations.

Chapter 3 (Semi Analytical Model) elaborates the theory behind the formation of the engineering problem in terms of the acoustic wave equations through the system under consideration. In the second part, a general solution has been derived analytically. It has been shown how the particular solutions can be obtained using both analytical and semi analytical methods. Limitations of this technique have been mentioned under the summary.

Chapter 4 (Numerical Model) details the methodology of applying numerical solvers used in MATLAB for determining the general problem and its solution. Within the Transfer Matrix Method (TMM) approach the losses between the rings have been accounted for by directly considering sound propagation in slit like pores and by using equivalent fluid model. The results of these have been compared. The most reliable version of these numerical techniques has been used to analyze the concept.

Chapter 5 (Finite Element Method Model) is although based on a similar approach, yet the numerical approach using Finite Element Analysis forms a separate chapter. In this chapter the engineering problem has been modelled in the Multiphysics solver COMSOL by defining the geometry and associated variables and boundary conditions. The solutions have been processed and results have been obtained in the post processing stage of this approach. In addition, certain variables have been parameterized for generating comparative studies to optimize the design of the concept.

Chapter 6 (Results and Discussion) describes the investigation of the various approaches and presents the observations. The characteristics which demonstrate the functionality of the muffler have been discussed. Comparative analyses amongst varying parameters have been studied and detailed in this section.

Chapter 7 (Conclusion and Future Scope) summarizes the conclusions from the present study and also hints at the possible roadmap for future investigations.

Chapter 2

Review of Literature

2.1. Sound Attenuation

2.1.1. Introduction to Sound Attenuation

As an acoustic wave propagates through an open ended system, generally some portion of the energy is reflected while some gets transmitted through it. Many-a-times, a third part of the energy gets absorbed within the system, which then gets accumulated and dissipates as heat. Thus absorption of sound is a process that a through-propagating acoustic wave encounters and the degree of which depends on the response of materials involved in the system and the nature of media it encloses. In real engineering problems, there is always certain amount of visco-thermal energy loss and scattering within the systems resulting in attenuation or damping of sound. In many situations, acoustic attenuation is considered a boon that serves as a key acoustic treatment technique for several engineering fields.

2.1.2. Basic Properties

Acoustic attenuation within a system can be characterized by certain terms that convey the physical significance of its impact. The main terms used throughout the analyses of this study are briefly reviewed as follows;

- <u>Impedance and Admittance</u>

For an acoustic wave travelling through a system, surface discontinuities and changing nature of media act as hindrances to its normal passage. In scientific parlance, these obstructions are defined in terms of impedance and admittance of the system, wherein impedance and admittance would be antonymous for particular references.

The impedance for the propagating sound wave is the ratio between the acoustic pressure of the wave and acoustic mass velocity. When used generally, acoustic impedance represents the characteristic impedance which are the impediments offered by the structure of the system. The other form of impedance is the radiation impedance which is imposed by the ambience or environment onto the acoustic radiation from the termination of the open system in consideration. In many cases, the impedance is a complex expression consisting of a real and an imaginary part. The real part contributes to the acoustic resistance that mainly occurs due to the friction offered by the medium while the imaginary part is the acoustic reactance due to the inertia and elasticity.

While impedance helps in determining the reaction of the medium to the transmission of the acoustic wave through it, the admittance determines the acoustic mobility, which is the inverse of impedance. Admittance is the ratio of the volume velocity to the acoustic pressure of the wave. However, both these terms are useful in describing the acoustic radiation.

- <u>Coefficients</u>

The behaviour of propagating acoustic waves within a system in response to its structure and media are expressed in terms of coefficients;

  - Reflection Coefficient
  - Transmission Coefficient
  - Absorption Coefficient

The reflection coefficient relating to the reflectance of the system is the ratio of the reflected to the incident wave pressure. The transmission coefficient measures the amount of transmission of the wave with respect to its incidence. The part of the wave that has neither been reflected nor been transmitted out of the system is usually considered to be absorbed and a comparison of its quantity with the incident pressure is the absorption coefficient.

For most open termination classical problems, squares of absolute values of these coefficients sum up to unity. In cases where there is closed termination, understandably, the transmission term is absent. In these systems the end can be either a closed structure (dead end) or rigidly backed by same or different material wall. In special cases, which involve pure non-dissipative scattering and non-relativistic mechanics problems, the absorption coefficient is not accounted for and the system behaves as per the reciprocity principle.

- Anechoic Termination and Non-reflecting Boundary

Anechoic, literally speaking is non-echoing or non-reflecting condition when associated to terminations makes a system to possess purely forward travelling waves with no terminal reflections at interfaces whatsoever. It can also represent an infinite termination. It is here when the equivalent impedance equals the characteristic impedance.

Non reflecting boundaries can be classified as matched boundary and radiation boundary. In case of the matched class the boundary is considered to be at infinity while for the radiating class, it assumes the presence of an outgoing wave in addition to an incoming wave.

- Transmission loss and Noise reduction

Transmission loss is the loss in the power levels when an acoustic wave is transmitted through a muffling section within a system. Its calculation is independent of the acoustic source and acts as an important performance characteristic of the muffling unit. The primary requirement for measuring the loss is the presence of an anechoic termination at the downstream end. Noise Reduction or Level Difference is defined as the difference in the sound pressure levels at two arbitrarily probed positions. It does not require anechoic termination like transmission loss; however, if the termination were considered as anechoic then noise reduction would be treated as a limiting situation of transmission loss for that muffling section.

- <u>Insertion Loss</u>

Besides the transmission loss and noise reduction, insertion loss is yet another parameter that helps in rating of the performance of a muffling section. The difference in radiation power level obtained when the system is compared in the absence and presence of the acoustic filter is called the insertion loss. As defined, it specifies the direct impact of the presence of a muffling unit in between the source and the receiver. Of the various parameters, the insertion loss is treated as relatively more significant as a criterion for analyzing the performance of the muffler.

- <u>Propagation velocity – particle velocity, phase velocity and group velocity</u>

The velocity of a particle of the medium relative to entire medium produced by a moving acoustic wave is particle velocity. Wherever needed, it can be associated to acoustic volume or acoustic mass velocity.

The summation of the flow velocity to the sound velocity, when medium is at rest, equals the phase velocity. It determines the rate at which the phase of the sound wave is moving. The group velocity is that with which the overall shape of wave amplitudes is moving. Its direction of propagation is different from that of the phase velocity. But in cases when the wave is moving parallel or anti-parallel to the flow, is when they are aligned. When a number of plane waves having different wavelengths and propagation velocities move, dispersion occurs forming wave packets that scatter all over. This phenomenon is closely linked to the phase and group velocities.

2.1.3. Sound ‘muffling and silencing’

Unwanted or unappreciated sound, often classified as noise, in most instances is attenuated by either ‘muffling’ it or ‘silencing’ it. Literally speaking, the term ‘muffling’ suggests the use of wrapping, blocking or insulating materials that repress the generated noise. ‘Silencing’ on the other hand, implies the process of muting or deadening of the produced sound. Although, in terms of the sound attenuating devices, the muffler and the silencer is often used interchangeably due to the common purpose they contribute towards, yet, there exist certain distinguishable features that differentiate them from each other. These differences are primarily in terms of morphology and in turn their mechanisms of dampening the undesired sound.

2.2. Mufflers and Silencers

2.2.1. Introduction

Dating back to the late 1890’s, was the period when the need for designing and analyzing muffling devices arose (Reeves, 1897). Later, as people grew increasingly aware of the environment they dwelled in, the curiosity of application of muffler and silencers began to thrive. Since then, the national agencies and governments have introduced and set to practice noise control rules, regulations and acts.

Majorly targeting the modes of commutation including the roadways, railways, airways and even waterways, every sector of transport has been categorized under acceptable noise emission levels. Each of these modes, such as, off-highway and on-highway automotive vehicles, locomotives, aircrafts and over- and under-water conveyances have been developed with an inclusive muffling technology as per the noise sources within them.

Following this, the present day sees an increasing applicability of muffling systems in factories and built environment. Internal noise reduction through ducts, vents and pipelines as also within smaller applications such as the domestic appliances, experience a rising need of muffling and silencing advancements.

2.2.2. Description

A muffler or silencer (generally used interchangeably) is a low pass filtering device that restricts the passage of the unacceptable part of sound and allows free access to the desired part, as also to the free flow of fluids, where present. Other synonymous variants of the sound attenuating devices, of which the muffler or silencer either form a subset or act as a whole unit, can be enlisted to include all but not limit to absorbers, expansion chambers, catalytic convertors, particulate filters, exhaust gas processors, sound suppressors and moderators. These devices are located either in close proximity to the source of sound or act as an essential element along the transmission path, generally placed somewhere half way through or towards the trailing end of the downstream termination. Their positioning is specific to the targets of attenuation, be it general purpose noise reduction or locally focused damping. Several idealizations have been proposed throughout literature to realize the design of potential muffling devices.

According to Pierce (Pierce, 1995), within an idealized muffler configuration with the source characterized by exhaust volume velocity, the frequency based components of the velocity must not vary. The ideal assumptions that govern any given muffler design are enlisted as hereunder;

1. Each frequency component propagates independently due to which the transmission characteristics are analyzed in the frequency domain and other frequency related effects are also accounted for.

2. The spectral density of the volume velocity injected into the exhaust system must not be affected by the introduction of muffling elements within the pipe configuration.

3. Acoustic wave interaction with the mean exhaust flow must not be considered.

The pre-design phase of any muffling or silencing device is critical owing to the simultaneous satisfaction of a combination of conflicting demands, associated directly or indirectly to the acoustic pressure wave propagation; the criteria of which have been briefed (Beranek, 1971):

1. Acoustical Criterion: Acoustic performance of the system is highly dependent upon the operating conditions and the nature of the source, mainly the inflow volume velocity. Also as its primary function, the minimum noise attenuation to be achieved is an important specification.
2. Aerodynamic Criterion: Based on the area of application, any system has a defined range of operating conditions in terms of the working temperature and mass flow rates. This criterion indicates the average pressure drop requirement subject to the system conditions. This information aids in analyzing the attenuation capability of the muffling section.
3. Geometrical Criterion: Focusing on the customer requirements, this criterion lays restriction on the system dimensions that are specified as the maximum allowable internal volume and overall device shape.

4. Mechanical Criterion: In order to sustain the muffler's efficacy and robustness, the mechanical criterion lays stress on the use of suitable materials and its overall structural design.
5. Economic Criterion: Last but not the least, the key parameter of incurred costs associated with the system form an essential parameter. In the process of satisfying all of the above criteria, the economics, including that for manufacturing, installation, operation and maintenance, is accounted for in this criterion.

The broad specification of the essential and desired design criteria aid in determining the design requirements and thus narrow down the parameters during the design phase. As put together (Munjal, 2014), the generic design requirements of any muffler or silencer (focusing on automotive applications, in this case) are enlisted as follows;

1. Adequate insertion loss: The minimum expected noise from a system in the presence of muffling or silencing units should be a few dBs lower than the most predominant noise sources through the system.
2. Back pressure: The insertion of a muffling device in the normal transmission path of the exhaust or flow exerts an extra static pressure on to the system, say the engine in case of automotive devices. In order to maintain less burden onto the driving mechanism, the additional component of muffler induced back pressure must be minimal.
3. Size: To adjust with the overall room space, weight and cost considerations of an added device such as a muffler, the size should be appropriately selected.
4. Durability: The ability to resist corrosive and resistive internal environments as also the impact of high exhaust gas temperatures, the sturdiness of the muffling device must be well thought-out.

5. Quality of sound: The sound emanating from the system after passing through the muffling unit must lie towards the lower end of the range of normal audible tolerance limits. For instance, in case of automotives or other domestic appliances fitted with mufflers, the sound quality at the start and during the operation must not be harsh enough and the functionality of silencing must be effective.
6. Muffler performance: The device performance must not deteriorate with the passage of time. The internal components must be opted for so as to assure a long life time of the device as a whole.
7. Flow generated noise: Devices with the expectation of higher insertion loss tend to also contribute to flow generated noise. The mufflers must however, be such that the generation of such noise should be kept under check.

Based on the requirements as detailed above, manufacturers establish specific guidelines for ideal characterization of their muffler based products. These can be classified as;

1. External Characteristics: The external morphology of muffler variants that are presented in the product guides highlight the following features:
    a. Shape: Mufflers or silencers can be designed in several cross sections such as Round, Oval and Square Box shape models.
    b. Style: There are several configurations in terms of the inlet and tail pipe orientations, which include the elbows and bends. They impact on the inflow of the fluids and thus the performance requirements of the muffling unit.
    c. Materials: To take care of the energy exchanges in terms of vibrations and temperature, the material of the external surfaces is essential. It also considers the overall weight and cost of the system.

2. Internal Characteristics: The main function of the muffler or silencer is governed by the internal components. They are further categorized as;
    a. Insulation: Internally if there is an insulator wrapped or not, during the process of canning of a muffler it has an impact on muffling characteristics.
    b. Insulation application: In case the insulator is present then the functionality that it accounts for, be it, focusing on the sound attenuation or insulation against dissipating heat, is a characteristic of concern.
    c. Internal components: Besides the presence of an insulator, sound attenuation or basic muffling and silencing demand the presence of additional components to keep up with the growing market competition. Thus the quantity, size, position and orientation of internal baffles, tubes and catalysts are by far a part of the major internal characteristic governing the product classification.

2.2.3. Acoustics of Mufflers

Although fundamentally speaking, any muffling or silencing device solves the single purpose of sound attenuation, yet when it comes to catering to the needs of specific domains of application, there can be several observable variations among these devices. Conventionally, there are two broad heads under which mufflers can be classified;

1. Passive Mufflers
2. Active Mufflers.

However, the most widely researched upon and in practice is the Passive type. These mufflers are further sub categorized into three concepts;

1. Reactive Mufflers

2. Absorptive Mufflers
3. Hybrid Mufflers, which are also a combination of reactive and absorptive types.

Each of these sub types are detailed as follows;

Reactive Mufflers:

Muffling devices that prevent or strongly reduce the propagation of acoustic waves by reflection or suppression due to the internally present discontinuities are called reactive mufflers. Dissipation effects can be neglected in such devices. Their inclusion within a noisy system has a powerful impact on the generation of sound at the source. The discontinuities play an important role in altering the acoustic impedances of the devices that contribute to the reduction in the sound power. In an ideal condition, no power would pass through when the acoustic impedance is negligible. Thus the impedances of the source and at the termination influence the performance of the reactive device (Pierce, 1995; Bies and Hansen, 2003)

Bies and Hansen (Bies and Hansen, 2003) has stressed on the impact of discontinuity by specifying that these are nothing but the contractions, expansions, bends and other such changes in surfaces and media present inside the muffling unit. These majorly contribute to the dynamic losses within the system. In practical cases, the sound wave additionally encounters frictional losses while traversing through the length of the muffling section. The summation of frictional and dynamic losses together constitutes the overall pressure drop of the muffling system which is imposed onto the source or the driving unit.

Munjal (2014) has reported that the configurations of reactive mufflers, essentially consist of a combination of several tubular and plate like elements of different transverse dimensions. Thus internally, at every junction there is a discontinuity created causing an impedance mismatch in turn leading to reflection of most of the incident wave back to the source.

In simple words, Pierce, 1995 has explained the basic mechanism by which reactive or reflective mufflers function. Plane waves entering a muffling device interact with the discontinuities on their path of propagation in such a way that they undergo perfect reflection. Despite the presence of insignificant attenuation in the upstream direction of the muffler, the reflections caused by the discontinuities produce standing waves within the device. These standing waves have a pressure at the source which is 90° out of phase with the source volume velocity. This reduces the acoustic power radiated off the outlet.

For these structure-based reactive muffling devices, their analyses are based on acoustical analogies of the well known Kirchoffs laws, as put forth by (Bies and Hansen, 2003), stating:

1. The algebraic sum of acoustic volume velocities at any instant and at any location in the system must be equal to zero.
2. The algebraic sum of the acoustic pressure drops around any closed loop in the system at any instant must be equal to zero.

Of the several available reactive muffler designs, the basic variations can be classified under the following generic constructions (Munjal, 2014; Bies and Hansen, 2003);

<u>Expansion Chamber:</u> (Middelberg, 2004) Perhaps, the simplest version of a fully functional reactive muffler is the simple expansion chamber. With chamber cross section to be less than half wavelength in dimension in order to neglect the effects of propagation of the wave, these mufflers form the key elements that can be placed in line with the pipe configurations.

Low Pass Filter: (Yasuda et al., 2013) An acoustic filter is a device that comprises of a set of acoustic elements between the source and the receiver. Such a device is commonly used for the suppression of pressure waves in flowing media within the muffling arrangement. They represent exhaust mufflers with acoustic waves being convected downstream by the medium in motion and can be termed as Aeroacoustic filters. However, in theory they are analyzed by considering plane wave propagation through stationary medium. These filters are analogous to electrical filter and vibration isolator so much so that that they are sometimes referred to as acoustic transmission line. Low Pass acoustic filters are also called Helmholtz filters. A low pass filter functions well at low frequencies where the element dimensions are comparable or lesser that half wavelength long. At this frequency range the construction lose their significance and the muffling elements can be analyzed by representing them as an equivalent acoustical circuit. The analysis of these filters cannot be carried out in isolated and must include the effects of the impedances of the other elements coupled along with that play a major role in the overall performance of the muffler.

Side Branch Resonator: (Snyder, n.d.) A reactive device consists of a side branch to its main duct such that the branch resembles a short length of around quarter wave stub. The Helmholtz Resonator, which consists of a connecting orifice and the backing volume bulge, can be considered as another representation of this resonator. Targeting the suppression of mainly the pure tones that have constant frequency, the side branch resonators work to their optimum efficiency when their internal resistance is sufficiently low and they are situated in-line to the real part of the tonal impedance that is considered for suppressing. Parallel alignment of very low impedance with respect to that of the rest of the elements of the main pipe is the main principle of the functioning of this resonator.

Based on its functioning, these mufflers are used for sound attenuation in constant speed pumps or blowers.

Resonant Mufflers: (Honda Motor Co., Ltd., 2012) Based on the knowledge of the side branch resonator that can suppress local pure tones, the resonant mufflers can be described as the mufflers that comprise of a smart combination of quarter wave tubes and Helmholtz Resonators. These mufflers are tuned to resonate at the desired frequency range. They are appended onto the walls of the pipes or ducts of the systems through which the acoustic waves are due to propagate. As they act as one of the elements of sound attenuation within the entire duct configuration, there may be instances when these muffling sections are included with dissipative material for enhanced overall performance. However, in most cases they are used without the sound absorbing materials and essentially in applications where the porous sound absorbing materials cannot be used, say in situations where the use of materials risk the contamination of the gas flow or vice versa. Resonant Mufflers find applications industrially where large low frequency sound reduction is highly demanded.

Absorptive Mufflers:

Unlike the reactive mufflers, the absorptive mufflers do not depend on the sound power alteration to attenuate it. Instead, they dissipate the sound energy that has entered or is generated within the system before it escapes the muffler. Mufflers that are governed by dissipative effects are termed as dissipative or absorptive mufflers. These mufflers also do not depend on the inclusion of discontinuities and thus, they tend to impose lesser overall pressure drop on to the driving system, in comparison to the reactive type of mufflers.

The mechanism of functioning is much simpler in the case of absorptive mufflers. Plane waves that travel through a muffling unit undergo attenuation of its amplitude under the strong influence of presence of an internal lining. The lining is absorptive in nature and the attenuation takes place without appreciable reflection or alteration of the ratio of sound pressure to source volume velocity. The amount of attenuation that can be achieved through a lined segment depends on the capability of the acoustically absorptive material, which depends on the material, quantity, thickness and positioning within the muffling system. The energy that gets dissipated from the system and absorbed within the porous liner travels through the liner and escapes as unwanted sound waves along with the induced draft.

Generally these porous lining materials are provided with facing or wrapped with insulation to protect them from getting damaged due to the draft velocities or the thermal influence of the dissipated energy. The liner must also be such that it should incorporate some undamped resistance in the form of a simple hole through which or across which some flow is induced.

According to (Munjal, 2014), the analysis of an acoustic wave propagating through a simple muffling system such as a duct internally lined with an absorbent material, can be carried out in the following two approaches;

1. Locally Responding Liners: This approach is traditional and well established and simplified to ease the process (Morse, Ingard and Beyer, 1969). The assumption that the absorbent lining is locally reacting implies that the liner can be treated as if the local impedance characteristics are independent of any other portion of the liner. In addition, this approach also assumes that the sound wave propagate in no other direction but normal to the surface.
2. Bulk Responding Liners: (Munjal and Thawani, 1997) The Bulk approach differs from the local approach as the sound wave propagation is considered to propagate

not merely in the normal direction but also in the direction along the surface. The sound propagation within the liner is also accounted for. The treatment is not as easy as that of the local case, however in order to study the impact of the liner on the attenuation of the sound traversing through the muffling system, it is assumed that the liner duct is in continuation with the main duct. In this case the sound through the liner is subjected to an effective sudden expansion of the cross section at the inlet of the duct. Quite similar in operation to the simple expansion chamber, as introduced within the reacting muffler classification, the sudden expansion affects the acoustic waves in an analogous manner.

Although the basic functioning of the absorptive muffler is such that besides the material properties there is hardly any method of classification, yet there can be certain variants within this muffling type.

<u>Diffusers as Dissipative Mufflers:</u> (Beis and Hansen, 2003) suggest that based on their application in the high pressure exhaust gas noise reduction, the gas diffusers act as dissipative mufflers within these systems. The main functions of these diffusers that can be associated with the muffling characteristics are:

1. The pressure gradient built up within the exhaust must be primarily reduced.
2. The shear in the mixing region where the exhaust gas and the ambient air assimilate must be controlled.
3. The shock waves generated by the exhaust flow must be stabilized.

<u>Lined Plenum Attenuator:</u> (Spon, 2004) Plenum is an enclosure having higher air pressure over its surrounding. When such an enclosed space is internally lined with absorptive layer, the fluctuations of air pressure can be smoothened out. Along with the air pressure, the acoustic wave based pressure fluctuation can also be attenuated. Thus in applications

such as the air conditioning systems, lined plenum type of muffling systems have been installed.

Thus, dissipative type of mufflers find applications in the muffling or silencing sound in fan and air conditioning systems as also certain draft induced systems as discussed above. However, there is a strong reason to this as well. According to (Munjal, 2014) if pure dissipative mufflers are used for automotive applications such as vehicular engines, there is a high risk on the longevity and performance. With time, the pores and these acoustic linings get congested with the exhaust gas species and may undergo thermal cracking. Thus, in automotive applications, they are getting replaced by reactive mufflers on account of:

1. Clogging of the unburnt carbon particles within the pores of the acoustic absorptive materials
2. Fibers being blown out from the lining due to the high velocity unsteady flow of the exhaust gases
3. Thermal cracking and other forms of thermal damage of the lining material
4. Poor attenuation at low frequencies, of the order of the exhaust noise
5. Comparatively high material costs in addition to the maintenance costs of these muffling systems

In case of application areas such as the ventilation systems, air conditioning systems, industrial fans, access openings of acoustical enclosures, intake and exhaust ducts of power stations, cooling tower establishments, turbines and jet engine test cells, unlike the automotive systems, the air that is being circulated is clean and bearable temperatures. The noise propagating through the air handling units and other duct based systems are reduced not only by the means of impedance mismatching but majorly by the internal lining provided within the ducts. A definite attenuation in the noise levels is observed for a wide range of frequencies. Finer applications where there is no flow involved such as the slits of doors and concepts such as the anti-noise windows also make use of these mufflers.

However, modern research also suggests that there is an ongoing improvement in the fibrous materials that are being developed to serve various purposes. Sintered metal composites that resist clogging and thermal cracking and yet are cost effective, materials like long strand glass fibers that can withstand high temperatures and the likes are been developed to overcome issues and make the dissipative mufflers applicable in many more fields.

Hybrid Mufflers

Despite the elaboration on the clear demarcation of the two broad classes of mufflers, along with their advantages and limitations, a practical industrial muffler is always a combination with some elements providing impedance matching while the others causing acoustic dissipation. It is hard to create pure reflective or completely dissipative muffler, which will be highly effective in its target zone. Thus, for modern day muffling technology, however, engineering designers are intentionally popularizing the so-called Hybrid Muffling technology. This essentially mixes the best effects of both the classes of mufflers and creates highly efficacious products that nearly achieve the desired attenuation.

Active Mufflers

Although the design and analysis of passive mufflers have been under extensive research since long, yet with increasing regulations and customer specifications, certain insufficiencies have always been experienced, be it their poor performances at particular frequency bands or exorbitant prices. The drawbacks of passive mufflers have introduced the concept of active attenuation of sound.

A typical active noise control system within a duct functions based on the mechanism as described. The active approach of the sound attenuation technique is based on the feedback mechanism that many electrical / electronic governed systems reply on. Within this approach, the unwanted noise component of the exhaust is sensed and an adaptive inverted signal is fed back into the duct after being generated through an auxiliary source such as a loudspeaker. The original source and the feedback source then act as an acoustic couple that undergo destructive interference thus cancelling out each other's velocity output. Here it is the velocity output that is considered rather than the steady state power output which is zero (Munjal, 2014).

As it has been long realized that the displacement of the electrical signals are faster that the acoustic signals, the active muffling techniques are gaining attention. These mufflers do not exert back pressure and unlike the passive mufflers are less bulky and the overall costs can be recovered over a long period of time. However, this concept is still in the nascent stage of development and will have to overcome several complications and concerns, especially in terms of the sophisticated hardware support that its functioning would demand.

2.2.4. Summary

During the early 1900s much emphasis was laid on in researching upon the impact of shape and type variations for optimizing muffler designs. Some of the early patents (Beeching, 1965; Eriksson, 1981; Ford Motor Co, 1971; Mantyla, 2005; Schmidt, 1918; Sterling, 2001; Stonestreet, 1959; Visnapuu and Lay, 1978) holding vital information have germinated as major study areas in recent times. A few of the patents (Sheet Delivery, 1887; Schnell, 1931) described the impact of the shapes of outer shells of mufflers. Some of them elaborate the effect of internal inclusions such as the ring like structures and slotted plates and baffles.

In the present times, it has been expressed that there are three ways in which the muffling or silencing devices are expected to function, either individually or in their best combination; suppressing of the emanating noise, attenuation of the generated noise and finally the manipulate and drift away the harmful unwanted noise from the susceptible areas. Based on the types of muffling techniques classified above, (Bies and Hansen, 2003) suggest a quick reference table highlighting the most prominent mufflers of all.

In practical situations, it can be observed that for applications where the low frequency noise is to be attenuated then it is the reactive type of muffling system that is preferred. This is so because it is the reflective type that are more robust and compact over the other types. Thus, despite the limitations in terms of higher back pressure they are commonly used in internal combustion engines and other such demanding areas. Nevertheless, the dissipative or the absorptive type of mufflers is recommended for high frequency and broad range frequency noise attenuation applications. In order to be effective for such fields, these muffler types are considered as easy and cost effective in construction, operation and maintenance.

Based on the existing knowledge and the ongoing research, futuristic high performance mufflers could be realized as a hybrid combination of passive components for high frequency sound attenuation and active components for the low frequency attenuation (Munjal and Eriksson, 1989).

2.3. Muffling Mechanisms

With regards to the categorical classification reviewing the types of mufflers in the previous section, this section highlights, in brief, the most prominent passive muffling mechanisms.

- Impedance matching

When sound is transferred from one medium to another, an impedance matching occurs at the interface. If the acoustic impedance of the adjacent media are different, most of the sound energy is majorly reflected, and in some cases absorbed, rather than being transmitted across the interface. This forms the basis of an impedance matching problem. This technique is quite commonly used in ultrasonics and in the development of horn and loudspeakers (Bängtsson, Noreland and Berggren, 2003).

- Resonant absorption

The Helmholtz resonator and the membrane absorber, as discussed above, are the main types of resonant absorptive muffling mechanisms. These absorbers can be represented as spring-mass-damper systems as a simplification to arrive at meaningful outcomes. In case of the Helmholtz resonator (Wolfe, n.d.), the mass is the air column trapped in the opening while for a panel or membrane based absorber (Oldfield, 2006), the membrane material itself serves as the vibrating mass. The damping is provided by the inclusion of a small amount of absorbing material such as mineral wool which is necessary to take away the noise off the system.

- Porous Absorption

Absorption using porous materials is governed primarily by the nature and composition of the material (NASA/TM—216995, 2011; Leclaire et al., 2015). According to K.

Attenborough and O. Umnova (Wright, 2005) these materials can be classified into three major categories, namely;

1. Fibrous – which include absorbers in the form of mats, boards, performed elements manufactured of glass, mineral or organic fibers (natural or man-made), including the felts and felted textiles
2. Cellular – which are polymer foams possessing varying rigidity
3. Granular – such as the naturally occurring materials like sand, gravel, soil and snow

In order to visualize the mechanism of porous material based absorption, the materials as enlisted above can be considered as dissipative fluids. Despite their consideration in either consolidated or loose form, the acoustic wave motion through these materials is limited to the fluid within the pores. The acoustic pressure amplitude and phase get altered under the influence of the visco-thermal exchanges within the pores of the material (Leclaire et al., 2015).

- <u>Acoustic metamaterials -</u>

Configurations comprising of quasi-periodic or periodic arrangement of repeating structures consisting of solids and fluids embedded within a base medium are featured as metamaterials. (Umnova and Zajamsek, 2012; Jiménez et al., 2016). Sound absorption using these systems are sometimes based on the Acoustic black hole effect (Krylov, 2014). The working mechanism can be understood by the interactions and the resulting scattering within the structure as also the local resonance provided by the individual cells. A prominent impact of the presence of such inclusions within a medium on a propagating acoustic wave is the band gap formation. These prevent the transmission of the phonons (unit of vibration or sound energy) through the material at a target frequency range. The physical and morphological nature of the inclusions has a strong influence on the frequency range within which the transmission can be blocked.

2.4. Approaches

The study of acoustics in most general view is considered to be a balanced confluence of physical phenomena in terms of generation, propagation and interaction together with the physiological response of perception. Several of its aspects can be studied by adopting various approaches based on the complexity of the problem and ease of extraction of the desired outcome.

2.4.1. Theoretical / Analytical:

In contrast to the experimental study, the mathematical representation of a physical problem under study is classified as a theoretical approach, a massive subset of which is the analytical, or aptly termed, classical approach based on Newtonian mechanics. In relation to acoustics, the methods such as ray tracing or geometric analysis are prominently considered. Here the engineering problem is formulated by the use of first principles, avoiding as much of approximations as possible. Mass and momentum conservation equations in addition to the state equation are majorly considered under the theoretical basis. The essential intent of this technique is to understand the propagation of a wave through the fluid domain followed by the overall impact of the superimposition of all such waves. This is an easy approach with high level of clarity while attempting to derive the effects. However, devoid of discretizations and simplifications, this method proves to be cumbersome to work out with for obtaining particular solutions. Not all of the real life applications can be represented mathematically as a single model owing to the coupling impact of several resulting phenomena occurring simultaneously. This limits the type and complexity of engineering problems that can be solved for by pure analytical approach.

2.4.2. Semi Analytical:

This is an intermediate approach that can be seen as a convenient transition between the theoretical and numerical approach. With the use of no or highly necessary approximation, the general problem is defined and sometimes, the derivation extends even up to the general solution. However, the numerical intervention takes place when the boundary conditions are applied and particular solutions are expected. This is one of the popular approaches to understand the behaviour of a physical model quickly and compare against a pure numerical approach. The approximations incorporated within this study are the Webster Horn Equation based simplification accompanied by the WKB approximation. While the Webster's equation helps in scaling down the dimensional consideration of the system, the WKB Approximation (developed by **W**entzel **K**ramers **B**rillouin and Jeffrey, which is similar to LG Method formulated by **L**ieuville and Green (Bender and Orszag, 2013)), is a widely used approximation for dealing with spatially varying coefficient based differential equations. Solutions to these can only be worked out by the intervention of some numerical support.

2.4.3. Numerical:

The numerical methods, unlike analytical approach, consider the overall behaviour of waves within a fluid volume. In order to account for real life applications, numerical approach introduces approximations that affects the accuracy of the problem but considers most of the key aspects. Essentially the numerical approach includes the discretization of the problem and the solution is obtained when subjected to a pre-developed program running in the background of the numerical solver. Governing equations developed based on several available concepts and approximations, lead to simplifications that represent the problem more realistically, and thus are beyond the scope of manual calculations. The numerical model that has been considered as a pert of this study is the Transfer matrix method. As a typical numerical scheme functions, within this method, the system is broken down into subsystems that interact with the adjacent systems and thus through such transfer the information about the field applied at the beginning of the system helps in deriving the information at its end. This class of approach is advantageous for the fact that they encompass a wide range of applications. They represent and tend to a problem in a complicated manner that is not only hard to compute but also larger the problem, larger is the modelling complexity.

Chapter 3

Semi Analytical Model

3.1 Introduction:

Mironov and Pislyakov have shown that total absorption of sound is possible through a terminating structure with gradually decreasing cross section and wall admittance as in Figure 3.1 (Refer Figure 2 in Mironov and Psylakov, 2002).

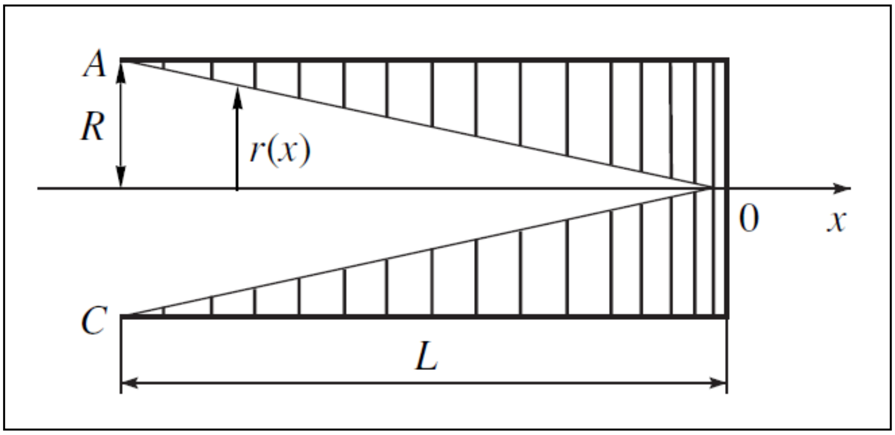

Figure 3.1: Figure 2 in Mironov, 2002 representing the acoustic black hole model

Following a similar approach, the wave equation for the semi analytical model has been developed by deriving the general Webster Equation for the open chamber design. For the low frequency acoustic waves propagating along rigid walled variable cross section tubes, the Webster Equation (Webster, 1919) considers 3D into 1D equation, expediting the analysis. The Webster equation is a linear differential equation with spatially varying coefficients, which can be easily solved for using the WKB Approximation within its validity to obtain general solution. The key assumptions that have been adopted are plane wave approximation and that the structure walls are perfectly rigid. The following sub section describes the model development of an open termination chamber design consisting of a flaring section which consists of varying wall admittance.

3.2 The Model:

The semi analytical model formulation for the open termination low frequency muffling system is as discussed in this chapter.

3.2.1. Wave equation in the tube with impeding walls

The derivation of the 1-D wave equation is governed by the general assumptions that consider plane wave propagation. According to the plane wave approximation principle, the range of validity has been considered $\sim R_0 < \frac{\lambda}{2}$. The solid hard boundary has been assumed to be representative of rigid walls.

Sound travelling through a medium propagates through it carrying along the generated pressure and velocity fluctuations. The wave equation for sound propagating through a simple tubular structure (see Figure 3.2.), is helpful in describing the behaviour of these fluctuations. It can be derived by using a combination of momentum equation, continuity equation and equation of state.

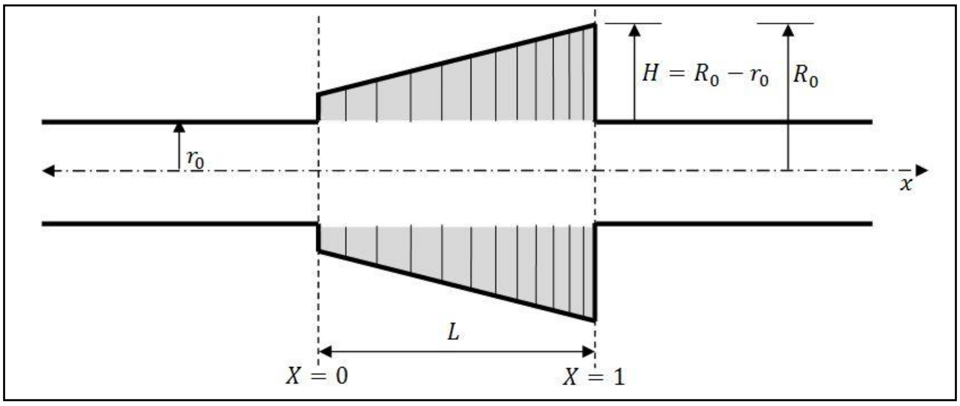

Figure 3.2: Open chamber design for low frequency muffler analysis

The Euler's equation, also known as the Momentum conservation equation, is a modified form of the Newton's II Law of Motion which relates the acoustic pressure and particle velocity in plane wave propagation, given as

$$-\frac{\partial p}{\partial x} = \rho_0 \frac{\partial v_x}{\partial t} \tag{3.1a}$$

where, p is the acoustic pressure, ρ_0 is the equilibrium density of medium and v_x is the axial component of particle velocity. This equation suggests that the particle's acceleration is directly dependent on the pressure gradient across the fluid particle considered within the geometry.

Conservation of mass based Continuity Equation is the relationship between the acoustic density and particle velocity. As per the mass conservation principle, if there is no gas flow present then the mass of the system remains constant. Considering a wave propagating along the *x-direction*, both term contributions, along and perpendicular to the direction of propagation have been accounted for. If the tubular section under study is a straight passage with no insertions or linings perpendicular to the flow then, the *y-direction* contribution can be ignored. However, with the presence of the rings as geometric insertions, their contribution has been added while calculating the effective cross sectional area. Thus,

$$-\frac{1}{\rho_0}\frac{d\rho}{dt} = v_x(ln\, S)' + \frac{dv_x}{dx} + \frac{2}{r}v_r \tag{3.1b}$$

where, ρ is the acoustic density, S is the effective cross sectional area, r is the tube section radius and $v_r = Y(x).p$ is the radial component of velocity and its relation with pressure such that the proportionality parameter $Y(x)$ is the wall admittance.

Equation of state also has to be taken into account;

$$\rho = \frac{1}{c^2} p \tag{3.1c}$$

where, c is the speed of sound in air.

Substituting Eq. (3.1c) in Eq. (3.1b), and simplifying, we get

$$-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = \rho_0 \frac{\partial V_x}{\partial t} \left(lnS(x)\right)'_x + \rho_0 \frac{\partial^2 V_x}{\partial x \partial t} + \frac{2Y(x)\rho_0}{r(x)}\frac{\partial p}{\partial t} \tag{3.1d}$$

Applying Eq. (3.1a) as necessary, and on rearranging,

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} + \frac{2Y(x)\rho_0}{r(x)}\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial x^2} + \frac{\partial p}{\partial x}\left(lnS(x)\right)'_x \tag{3.2}$$

Assuming harmonic time dependence in the form $p(x,t) = p(x)e^{-i\omega t}$, Eq. (3.2) can be rewritten as

$$p'' + p'(lnS)' + p(k_0^2 + i\omega \frac{2Y(x)\rho}{r}) = 0 \tag{3.3}$$

where $k_0 = \frac{\omega}{c}$ is the wave number in air with ω as the angular frequency. The prime and double prime denote the derivation with respect to x. The eq. (3.3) consists of the graded wall admittance that varies along x which has been accounted for by the presence of $Y(x)$. Cases with and without the inclusion of this variation have been considered later.

This is the general wave equation for tubular structure with impeding walls. It can also be referred to as the **generalized form of Webster's Equation**, which is a linear differential equation with spatially varying coefficients.

Here two important cases can be considered;

Case 1: If the $Y(x)$ is neglected, implying that the walls do not provide any impedance to the acoustic wave, then Eq. (3.3) becomes,

$$p" + p^{'}(lnS)' + k_0^2 p = 0 \tag{3.4a}$$

where this reduces to the conventional Webster's Equation with an impact devoid of admittance.

Case 2: If there is no variation of the tubular cross section, then $S = 0$ which reduces Eq. (3.3) to give,

$$p" + p(k_0^2 + \frac{2Y(x)\rho}{r}\ i\omega) = 0 \tag{3.4b}$$

3.2.2. Wall Admittance Derivation

An analytical approach to deriving the wall admittance $Y(x)$ has been briefed.

By definition, wall admittance is the ratio of particle velocity to the acoustic pressure.

$$Y = \frac{v}{p} = \frac{v_r}{dP} \tag{3.5a}$$

where, $v_r = -i\omega\, dr$ represents the radial component of angular velocity, dP is the acoustic background pressure fluctuation. Thus,

$$Y = -i\omega \frac{dr}{dP} \tag{3.5b}$$

Considering that the tubular section consists of a series of cylindrical segments, such that each segment comprises of a ring half and a cavity half part, the fluid volume occupied by this segment is,

$$V = (\pi R^2 - \pi r^2)L = \pi(R^2 - r^2)L \tag{3.5c}$$

where, V is the fluid volume occupied, R is the outermost radius corresponding to the cavity and r which is the tube radius is also the innermost radius of the ring. Now assuming adiabatic conditions, $PV^{\gamma} = constant$, where γ being the adiabatic constant, and $P = \rho_0 c^2$ is the background pressure, we get,

$$Y = -i\omega \frac{dr}{dP} = -\frac{i\omega}{\rho_0 c^2} \frac{(R^2 - r^2)}{2r} \tag{3.6}$$

This is the equation for wall admittance by involving the compressibility of the medium. The eq. (3.6) is the same as (eq. (3.5) obtained by Mironov and Psylakov, 2002) In order to stay within the range of applicability by avoiding the occurrence of resonance driven impacts, this eq. functions where $R \ll \lambda$.

Substituting Eq. (3.5c) in Eq. (3.3), and taking circular cross section, $S = \pi r^2$,

$$p'' + 2p'(\ln\ r)' + k_0^2 \frac{R^2}{r^2} p = 0 \quad (3.7)$$

This linear differential equation simplification of the Webster's Equation as obtained earlier, is applicable for wall admittance effect due to the variation of both the radii.

3.2.3. Normalized Equations

As observed, certain non-dimensionalities can be introduced within the equations derived above which in turn strengthens their applicability through parameterization and simplification. Carrying forward this normalization from the generalized form of Webster's Equation as derived in the Eq. (3.3), and to be more specific, considering the Case 2 or the constant S muffling section represented by the Eq. (3.4b), it can be simplified as,

$$\frac{d^2p}{dX^2} + pq_L^2\left(1 + \frac{2iY(x)}{q_r}\right) = 0 \quad (3.8)$$

Here, L is the overall length of chamber, $q_L = k_0 L$, $q_r = k_0 r$, represent the non dimensional frequencies with respect to length and radius respectively. Also x has been replaced by a dimensionless coordinate $X = x/L$ and Y has been normalized by the admittance of air. Let $\zeta(X) = 1 + \frac{2iY(X)}{q_r}$ be the dimensionless admittance function, then Eq. (3.8) takes the following form,

$$\frac{d^2p}{dX^2} + q_L^2\, \zeta(x)\, p = 0 \quad (3.9)$$

This Eq. (3.9) describes the pressure variation in the chamber with constant inner tube diameter and varying wall admittance, assuming plane wave propagation.

3.2.4. WKB Approximation

The linear differential Eq. (3.9) governing pressure variations in the muffling section, has a coefficient varying with coordinate X. To obtain a solution for such problems, the WKB Approximation, is a convenient analytical approach as evident from the literature reviewed. So as per the WKB Method, for a linear differential equation with spatially varying coefficients such as,

$$\varepsilon^2 f^n + a_1(x) f^{n-1} + \cdots + a_{n-1}(x) f^1 + a_n(x) f = 0 \tag{3.10}$$

has an asymptotic series expansion as its general solution which is of the form,

$$f(x) \sim \exp\left[\frac{1}{\delta} \sum_{n=0}^{\infty} \delta^n S_n(x)\right] \tag{3.11}$$

lying in the limit $\delta \to 0$ and S_n representing the arbitrary number of terms within the expansion series, essentially representing the phase functions, where, as per (Bender and Orszag, 2013) the initial few terms can be obtained as,

$$S_0'^2 = Q(x)$$

$$2S_0'S_1' + S_0'' = 0$$

$$2S_0'S_n' + S_{n-1}'' + \sum_{j=1}^{n-1} S_j'S_{n-j}' = 0, \qquad n \geq 2$$

Now representing the generic Eq. (3.10) into a particular format which is comparable to the Webster's Equation form,

$$\varepsilon^2 f" = Q(x) f \tag{3.12}$$

Comparing Eq. (3.9) with the general form of equation for WKB method application as expressed above in Eq. (3.12) where $Q(x) \neq 0$ and f corresponding to the pressure function, we have,

$$\begin{gathered} \varepsilon = \frac{1}{q_L} \\ Q(x) = \zeta(x) \end{gathered} \tag{3.13}$$

For $\varepsilon < 1$, the general solution comprising of complex coefficients, takes the form,

$$p \sim |Q|^{-\frac{1}{4}} \left[C_1\, exp\left(\frac{1}{\varepsilon^{\frac{1}{2}}} \int_a^x Q^{\frac{1}{2}}\, dx \right) + C_2 exp\left(-\frac{1}{\varepsilon^{\frac{1}{2}}} \int_a^x Q^{\frac{1}{2}} dx \right) \right] \tag{3.14}$$

where, C_1, C_2 are integration constants and are determined by applying the boundary conditions (Bender and Orszag, 2013).

3.2.5. WKB Conditions and Validity

The WKB approximation works within its limits of validity. Referring to the method, the conditions for applicability state that primarily as $\varepsilon < 1$ imply that $q_L > 1$.

In addition, the approximation is valid so long as the two conditions, $S_1 < q_L S_0$ and $S_1 < 1$, where S_0 and S_1 are 0^{th} and 1^{st} order terms, known as the Eikonal Equation and the Transport Equation, respectively, as intermediately obtained while deriving the general solution Eq.(14), are strictly satisfied. Evaluating these for the particular solution gives, $q_L > 1$, forming the lower limit for the zone of WKB Validity. As WKB renders the q_L set unbounded at the upper end, the plane wave approximation, fixes the situation. Accordingly, $q_R < \frac{\pi}{2}$ and since $q_L = q_R \left(\frac{L}{R}\right)$, using particular values, the upper boundary limit for WKB to be applicable for plane wave propagation, hence gets defined.

3.3. General and Particular Solution

To understand how the problem specific general solution would be formulated semi analytically, the WKB approximated method has been progressed with for a simplified condition.

Using dimensionless spatial variable, the variations of the outer chamber radius is described by the following function:

$$R(X) = r(1 + \alpha X) \tag{3.15}$$

where $\alpha = \bar{\alpha}L$ represents the slope of the flare. Now substituting in the Eq. (6) and hence obtaining the Eq. (13),

$$Q(x) = \zeta(X) = 1 + \frac{\rho_0 c\, k_w}{z_w k_0} \left((1 + \alpha X)^2 - 1\right) \tag{3.16}$$

If losses are neglected, i.e. $k_w = k_0$ and $z_w = \rho_0 c$ equation (16) is reduced to

$$Q(x) = (1 + \alpha X)^2 \tag{3.17}$$

Comparing with the Eq. (14) and considering lower orders of WKB, the general solution for this particular chamber design equates to,

$$p = (1 + \alpha X)^{-\frac{1}{2}} \left\{ C_1\, exp\left[iq_L X \left(1 + \frac{\alpha X}{2} \right) \right] + C_2 exp\left[-iq_L X \left(1 + \frac{\alpha X}{2} \right) \right] \right\} \qquad (3.18)$$

Let the inlet tube be treated as region I, the muffling chamber as region II and the outlet tube as region III. The interface between the regions I and II be section plane 0 and between regions II and III be section plane 1. It is at these interfaces planes that the boundary conditions have been applied to approach the particular solution. The equation of the plane waves propagating in the inlet and the outlet tubes are assumed to be based on the Helmholtz Equation while within the section it is as derived at Eq. (3.18).

Applying boundary conditions,

1. At $X = 0$, $p_I(0) = p_{II}(0)$ and $v_I(0) = v_{II}(0)$
2. At $X = 1$, $p_{II}(1) = p_{III}(1)$ and $v_{II}(1) = v_{III}(1)$

where, from the equation of conservation of momentum, $v = \frac{p'}{iqz_0}$. These four correlations generate four equations that can be expressed with the help or terms consisting of four unknowns and four variables. Being proportionate yet consisting of complex terms, they can be best solved simultaneously through inverse matrices by numerical tools such as MATLAB. The matrix elements are further utilized to obtain the acoustical characteristics such as transmission and absorption. This is what makes it a semi analytical approach, although most of the calculations can be simplified analytically.

3.4. Summary:

The Analytical Approach has been briefly introduced stating the analogy and the variation between Mironov's model and the open muffler model. The modelling technique has been sectioned into the wave equation algorithm, the WKB approximation and its zone of applicability. The main advantage of this technique lies in the mathematical and physical understanding of the engineering problem. It is helpful in identifying the parameters that describe the system and phenomena. The wave equation is in its most generic form which can be used to incorporate all of the flare shapes by substituting the corresponding waveguide functions. For convenience, the instance of a linear outer flare variation has been accounted for. In order to progress through the derivation to introduce the most widely used approximation, the case with absence of losses has been considered. However, when considering the presence of losses, the technique becomes cumbersome to apply. This can be identified as a major limitation of the semi analytical approach. This is yet another reason why a purely analytical approach must be necessarily worked out numerically. The other limitations include the idealism of theory and the high degree of assumptions to be considered. One such assumption is the reliance of the theory majorly on the plane wave approximation. This may render the condition of dimensions being much smaller than the wavelength to be not necessarily the sole considerations of sufficing its validity. Such concerns have been the key motivators for development of the model as discussed in the following section.

Chapter 4

Numerical Model

4.1 Introduction:

Realizing the muffling section numerically, the general problem formulation has been addressed by two methods. In order to compare the different approaches used so far, the Transfer Matrix Method (TMM) based approach has also been used to construct the engineering problem. The general and particular solution for the TMM based concept has been initially attempted by the use of Bessel functions, which has then been compared with and improved upon by the Johnson-Allard (JCAL) model.

4.2 Methodology:

The muffling chamber has been considered to be an array of repetitive units based on the number of internal rings as shown. (See Figure 4.1.) While Figure 3.2 provides a generic idea, flexible in terms of the flare dimension, this structure (Figure 4.1) particularly covers not only the external but also the internal flare variation.

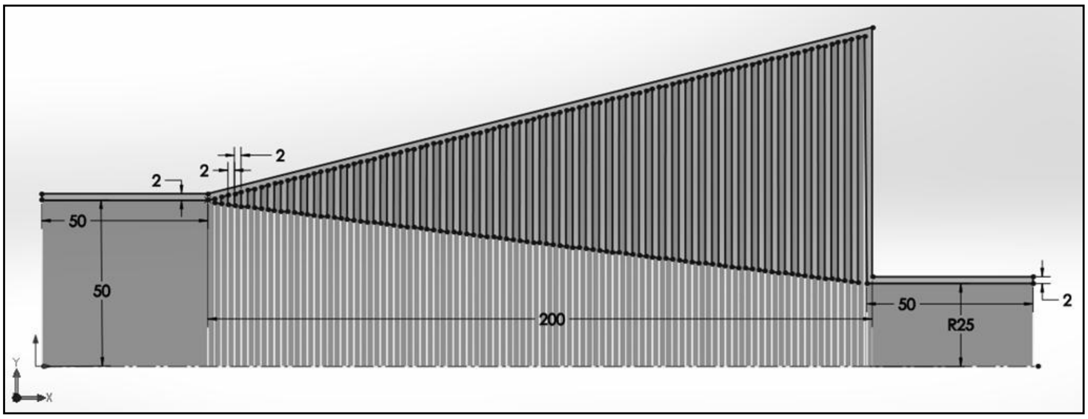

Figure 4.1: Geometry of structure shown for half cut section of muffler

Each unit is such that it is a union of the slit part and the ring part, together with its corresponding main cylindrical pore. The fluid volume occupied by the unit is essentially what defines the parts. Accordingly, the ring part of the unit comprises of the main pore segment while the slit region and its corresponding main pore segment has been further referred to as the cavity (Refer Figure 4.2). The internal and external radii and other geometric features such as the ring and slit thickness have been defined. For convenience in numerical formulation, the taper of each ring and slit combination have been assumed to be aligned with the main pore axis as seen in Figure 4.2. Transfer matrices have been constructed for each of the segments defined above, which are then combined as a unit and the matrix of the unit is varied spatially over the span of the entire muffling section to obtain the overall matrix. This consideration is similar to the one by (Guasch, Arnela and Sánchez-Martín, 2017) wherein, they consider the emsemble of cavity and ring for the close termination representation of the analytical model developed by (Mironov and Psylakov, 2002).

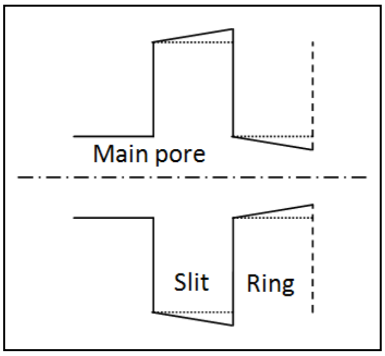

Figure 4.2: Singular unit for transfer matrix consideration

The propagation in annular rigidly backed slit pores is considered, which accounts for both viscous and thermal losses. In between the ring like structures, there is an effective medium with medium properties slightly different than the open passage that continues in line with the inlet and outlet tubes. The medium is treated as "effective" in order to account for the losses due to friction and thermal exchanges between air and the rings, which in the case of a simple tube would be trivial. The main parameters that would define the presence of this medium can be expressed in terms of the wave number and characteristic impedance between the walls. Let this effective wave number be k_w and the effective characteristic impedance z_w with the subscript w as an indicative of the influence of the ring wall. In an idealistic case of no losses consideration, these values would be equal to the corresponding quantities in air or the fluid medium through the system.

Similar to the previous method, here again it can be assumed that the system is formed as a union of several repetitive segments stacked in a linear array. However, for simplicity it is assumed that each segment could be represented by the Helmholtz resonator. Thus, in cylindrical coordinates and with no angle dependence, the Helmholtz equation for pressure between the rings is,

$$\partial_{rr}p + \frac{1}{r}\partial_r p + k_w^2 p = 0 \tag{4.1}$$

The radial component of the particle velocity, v_r is related to pressure by the momentum conservation equation that states,

$$-i\omega\rho_w v_r = -\partial_r p \rightarrow v_r = \frac{\partial_r p}{i\omega\rho_w} \tag{4.2}$$

where, $\rho_w = \frac{z_w k_w}{\omega}$ is the density of the wall medium, which is the effective density and would equate to the air or fluid density ρ_0, in the absence of losses.

The general solution of Eq. (4.1) in terms of Bessel Function representation is

$$p = AJ_0(k_w r) + BY_0(k_w r) \tag{4.3}$$

where, A and B are the integration constants and J_0, Y_0 are the Bessel functions of order zero. Now substituting Eq. (4.3) in Eq. (4.2) an expression for the radial component of the particle velocity can be obtained as,

$$v_r = \frac{k_w \left(AJ_0'(k_w r) + BY_0'(k_w r)\right)}{i\omega\rho_w}$$
$$= \frac{AJ_0'(k_w r) + BY_0'(k_w r)}{iz_w} \tag{4.4}$$

where the prime is derivative with respect to $k_w r$.

Considering boundary conditions in order to eliminate the integration constants, it is assumed that the ring has rigid or hard boundary at $r = R$ implying that, $v_r(R) = 0$ and using Eq. (4.4),

$$AJ_0'(k_w R) + BY_0'(k_w R) =$$
$$\rightarrow B = -A\frac{J_0'(k_w R)}{Y_0'(k_w R)} \tag{4.5}$$

Combining Eq. (4.5) with Eq. (4.1) and Eq. (4.3), the expressions for pressure and particle velocity in terms of single integration constant;

$$p = A\left(J_0(k_w r) - \frac{J_0'(k_w R)}{Y_0'(k_w R)}Y_0(k_w r)\right)$$
$$v_r = \frac{A}{iz_w}\left(J_0{}'(k_w r) - \frac{J_0'(k_w R)}{Y_0'(k_w R)}Y_0{}'(k_w r)\right) \tag{4.6}$$

The input admittance of a single cavity, the slit between adjacent rings, normalized to the admittance of air $\frac{1}{\rho_0 c}$ is

$$Y = \rho_0 c \left.\frac{v_r}{p}\right]_r \tag{4.7}$$

If the rings are infinitely thin, then the same expression will describe the wall admittance of the muffling section. If the rings are not thin enough, then there arises a need to account for the porosity of the walls which would be multiplied to the Eq. (4.7) to give the effective wall admittance. However, for simplification, assuming negligible ring thickness, substituting Eq. (4.6) in Eq. (4.7) and eliminating the integration constant,

$$Y = \frac{\rho_0 c}{i z_w} \frac{\left(J_0{}'(k_w r) - \frac{J_0'(k_w R)}{Y_0'(k_w R)} Y_0{}'(k_w r) \right)}{\left(J_0(k_w r) - \frac{J_0'(k_w R)}{Y_0'(k_w R)} Y_0(k_w r) \right)} \tag{4.8}$$

By applying Taylor's Expansions and assuming $k_w R \ll 1$ and $k_w r \ll 1$, Eq. (4.8) would direct to give Eq. (3.6). However, being more generic, considering cylindrical geometry and easy to account for the losses and resonances in cavity, Eq. (4.8) would be handier in the numerical analysis to be discussed in this chapter.

4.2.1 TMM model solved using Bessel functions

The transfer matrix of the unit constitutes the relation between the acoustic pressure to the particle velocity at both sides in the presence of the viscous and thermal exchanges.

For the slit part of the unit, the complex density governed by the viscous forces and the compressibility governed by the thermal exchanges have been individually derived and then combined to form the complex wave number and the impedance to account for the losses, as detailed for the analysis of slit-like pore.

$$\rho_w = \frac{\rho}{F_v} \tag{4.9a}$$

$$C = \gamma \left(1 - \frac{F_t(\gamma - 1)}{\gamma} \right) \tag{4.9b}$$

where, C is the complex compressibility, γ is the adiabatic constant and F_v and $F_t$are the viscous and thermal forces per unit volume of the element respectively (Wright, 2005).

Using Eq. (4.9a) and Eq. (4.9b), the complex wave number k_{slit} and the impedance z_{slit} are calculated as,

$$k_{slit} = k_0\sqrt{C\ \rho_w} \tag{4.10a}$$

$$z_{slit} = z_0\sqrt{\frac{\rho_w}{C}} \tag{4.10b}$$

The matrix characterized by the cavity has been derived as a combination of the lossy medium coupled with the influence of the normalized wall admittance as per Eq. (4.8). Thus accounting for the wall admittance in combination with complex parameters of the slits,

$$\gamma_{norm} = i\left(\frac{z_0}{z_{slit}}\right)\left(\frac{J_1(k_{slit}\ r) - \dfrac{J_1(k_{slit}\ R)}{H_1(k_{slit}\ R)}\ H_1(k_{slit}\ r)}{J_0(k_{slit}\ r) - \dfrac{J_1(k_{slit}\ R)}{H_1(k_{slit}\ R)}\ H_0(k_{slit}\ r)}\right) \tag{4.11}$$

The complex wave number and impedance of the entire cavity can be now obtained as,

$$k_{cav} = k_0\sqrt{1 + \frac{2i\gamma_{norm}}{k_0\ r}} \tag{4.12a}$$

$$z_{cav} = \frac{z_0}{\sqrt{1 + \dfrac{2i\gamma_{norm}}{k_0\ r}}} \tag{4.12b}$$

where, k_{cav} and z_{cav} – complex wave number and impedance of the cavity, respectively.

4.2.2 Introduction of JCAL Model in TMM

The cavity transmission matrix for a slit section (Munjal, 2014) can thus be formed as,

$$A_{cav} = \begin{bmatrix} cos(k_{cav}\,2x) & -iz_{cav}\,sin(k_{cav}\,2x) \\ -\dfrac{i\,sin(k_{cav}\,2x)}{z_{cav}} & cos(k_{cav}\,2x) \end{bmatrix} \tag{4.13}$$

The ring half of the unit, which is essentially the main pore, has been described by the Johnson-Champoux-Allard-Lafarge (JCAL) model (Wright, 2005) to account for the visco-inertial and the thermal losses.

$$\rho_w = \rho\alpha_\infty\left(1 + \frac{\sigma}{-i\omega\alpha_\infty\rho}\sqrt{1 + \frac{-i\omega}{\omega_b}}\right) \tag{4.14a}$$

$$C = \gamma - \frac{\gamma - 1}{1 + \dfrac{\eta}{-i\omega\sqrt{Pr}\rho k'}\sqrt{1 - i\sqrt{Pr}\left(\dfrac{\omega}{\omega'}\right)}} \tag{4.14b}$$

where, Pr is the Prandtl Number, σ, the air flow resistivity and α_∞, the tortuosity.

Here, ω_b and ω' are defined based on the parameters of the cylindrical pore geometry (Wright, 2005)

k_{ring} and z_{ring} have been formulated by substituting Eq. (4.14a) and Eq. (4.14b) into Eq. (4.10a) and Eq. (4.10b) respectively to obtain the ring transmission matrix for the ring section (Munjal, 2014) as,

$$A_{ring} = \begin{bmatrix} cos(k_{pore}\,h) & -iz_{pore}\,sin(k_{pore}\,h) \\ -\dfrac{i\,sin(k_{pore}\,h)}{z_{pore}} & cos(k_{pore}\,h) \end{bmatrix} \tag{4.15}$$

The TMM of the single unit is then calculated as a product of the ring and the cavity matrices obtained as

$$A = A_{ring} \times A_{cav} \tag{4.16}$$

Finally, combining this along with expansion and contraction matrices, as necessary, the transmission matrix for the unit is iterated to completely define the overall matrix of the model.

4.2.3 Expansion-Contraction Matrices

In addition to the overall transfer matrix characterizing the muffling section, certain chambers might have the presence of sharp area discontinuities in terms of angular bends in connecting tubes, sudden expanding and narrowing chamber edges or orifice based internal barriers or baffle plates. In these instances the overall TMM A, as obtained in eq. (4.16) has to be appended by additional matrices describing the expansion and/or contraction where necessary.

$$A_{cont} = \begin{bmatrix} 1 & 0 \\ 0 & \left(\frac{\max(r)^2}{R^2}\right) \end{bmatrix} \tag{4.17}$$

$$A_{exp} = \begin{bmatrix} 1 & 0 \\ 0 & \left(\frac{R^2}{\min(r)^2}\right) \end{bmatrix} \tag{4.18}$$

where, A_{cont} is the contraction matrix and A_{exp} represents the expansion matrix. Based on the values, the maximum and the minimum of the array have been chosen to form a part of the corresponding matrices.

As described by (Munjal, 2014) these physical surface changes can also be termed as the compliance, which represents a cavity that allows temporary acoustic energy storage and inertance, like an orifice of small length within a plate.

4.2.4 General Solution to TMM model

Following the generation of the final matrix, its elements are extracted for the calculation of the transmission loss, the coefficients and other parameters characterizing the performance of the muffling section under study; in the transmission regime and/or the rigid backed situation.

Considering $A_{11}, A_{12}, A_{21}, A_{22}$ as the elements of the final A matrix, in transmission regime, the reflection, transmission and absorption coefficients can be defined as,

$$RT = \left(\frac{\frac{t}{z_0 - 1}}{\frac{t}{z_0 + 1}}\right)$$

$$TT = \frac{1 + RT}{A_{11} + \left(\frac{A_{12}}{z_0}\right)} \tag{4.19}$$

$$AT = 1 - |RT|^2 - |TT|^2$$

where, $t = \frac{A_{11} + \left(\frac{A_{12}}{z_0}\right)}{A_{21} + \left(\frac{A_{22}}{z_0}\right)}$ is and intermediate simplification used and RT, TT and AT are the coefficients with T representing that the coefficients have been considered in the transmission regime.

Similarly, in the case of rigid termination, the equation set used for obtaining the reflection and absorption coefficients are as follows;

$$RR = \left(\frac{a_n - 1}{a_n + 1}\right) \tag{4.20}$$

$$AR = 1 - |RR|\text{^}2$$

where, $a_n = \frac{A_{11}}{\frac{A_{21}}{z_0}}$ is used for simplification and RR and AR are the coefficients with R as a suffix conveys that the coefficients have been calculated for the rigidly backed situation.

4.3 Summary:

The TMM based numerical modelling approach has been elaborated upon in this chapter. The description emphasizes the consideration of unitary sections of the entire muffler domain, which is further parted into the slit and the ring. The six independently measurable parameters (viz. Porosity, viscous and thermal permeability, tortuosity and viscous and thermal characteristic lengths) that the JCAL model depends upon have been derived. Using this model, the dynamic density and the bulk modulus have been obtained that form an essential part in the formulation of the complex valued impedance and wave number. Thus, in the presence of visco-inertial and thermal losses, these frequency dependent parameters aid in completely describing the propagation of sound waves through the muffling section. Once the final TMM has been defined, the acoustical properties such as the transmission loss and the reflection and absorption coefficients in the transmission regime can be obtained. The advantage of this technique lies in its close to practicality consideration in terms of loss inclusion and ease of analysis. MATLAB based coding, compiling, debugging and running is less tedious and time consuming. However, there still exist certain strong assumptions while defining the TMMs which render it still as an approximated form in terms of the morphology of the real model. In addition, numerical based solvers do not provide a visual representation of the model under study, making it slightly precarious as an approach. The 3D depiction concern has been well tackled with, in the following section, which also aids as a validation approach to the previous techniques.

Chapter 5

Finite Element Method Model

5.1 Introduction:

In order to have a Finite Element Method (FEM) based comparison to the WKB based analytical approach and the TMM based numerical approach, 3D COMSOL models have been developed for better visual understanding. The Pressure Acoustics Module within the COMSOL Multiphysics with MATLAB; versions 4.3b and 5.1 have been used mainly to generate and analyze the concepts. The studies have been carried out in the preset frequency domain that accounts for the most convenient representation of the results.

5.2 Processing Stages:

5.2.1 Pre Processing:

The fluid domain is modelled and meshed applied for physics of Pressure Acoustics within the Frequency Domain as specified earlier. Within the pre processing stage, as an initial step, the problem definition and the declaration of the associated parameters and variables is done. This is followed by the geometrical formation where the muffling section considered is mostly axi-symmetric, which is easy to construct. As an alternate approach a pre designed and/or meshed model can be imported in compatible formats from other software as well. However, care should be taken to consider that like most FEM based solvers; COMSOL design modelling is also based on the construction of the fluid domain rather than the external solid structure. The material and the medium is defined in this stage as well. The variables mainly consist of the characterizing entities that help in defining the nature of the material and medium, as also the expressions for the post processing. The inlet and outlet boundaries, generally, are considered as an explicit selection and then integrated over for applying the boundary conditions later.

5.2.2 Processing (Boundary Conditions):

The processing stage essentially is system driven, although the necessary settings in terms of the boundary conditions need to be pre set for the software to work on. Within the Pressure acoustics module, the defining of the following is crucial;

- Sound hard boundary, which comprises of the outer walls and internal structures that do not allow the fluid to travel through.
- The initialization of the pressure and velocity at the start has to be set to zero.
- At inlet, the incoming wave while at the outlet, the outgoing wave motion has to be set into the plane wave radiation definition where the incident acoustic field is applied to the geometry.
- The end domain of the geometry is declared as a perfectly matched layer to account for the non rigid or anechoic termination.

Following this the model is meshed based on the surfaces and dimensions of the smallest and the largest structures of the model. This is also governed by the nature of the acoustic wave incident through the fluid domain of the model. For instance, the size of the mesh has been defined in a particular case as;

Maximum element size $\frac{c}{8\,(\max f)}$ would account for a minimum of 8 mesh elements of the geometry for a particular wavelength of the acoustic wave. Similarly, the minimum element size $\frac{c}{30\,(\max f)}$ would help to determine that the user defined mesh is fine enough to capture the physics satisfactorily. The growth rate of the mesh is set to 1.2 to ensure that the meshing grows evenly from the edges inwards. As convenient, the tetrahedral and triangular mesh types have been made use of.

On setting the frequency range based on either the number of steps or the number of values, the code is set to study or process. A frequency range of 10 – 1k Hz has been considered to account for the low frequency range consideration as per the problem definition.

5.2.3 Post Processing:

Once COMSOL has completed the internal calculations, the processed information is available for the user to pre-process it. The plots for the acoustic pressure including the total and the scattered fields, the sound pressure levels, the local instantaneous velocities in the $-x$ and $-y$ axes and the global line plots for the transmission and absorption characteristics have been mainly derived.

The coefficients have been plotted against frequency by probing the pressure distribution averaged over the cross section at the interface between the tube and the chamber. Thus, while transmission coefficient corresponds to the absolute of the pressure intensity and the exit of the chamber, the reflection coefficient was calculated as the absolute of the difference between the pressure at the entrance and the incident pressure. The absorption coefficient was then calculated by eliminating the squared of the reflection and transmission coefficients from unity. The plots and graphs as obtained have been discussed in the following chapter.

5.3 Summary:

The 3D FEM solver based approach that has been discussed in this chapter has aided to provide a visual understanding of the problem. This approach has been used as a validation for the analytical and numerical models detailed earlier. The advantage of this method is that the problem statement can be defined by the GUI interface developed by the software. The model library has pre defined applications and materials that serve as a handy tool to start off and build on for general analysis. Realizing the model through this technique, the essentialities and precautions for the experimental setup can be established well in advance.

Chapter 6

Results and Discussion

6.1 Performance evaluation of muffler based on holistic consideration:

An overall impact on the performance of silencers and mufflers by varying its parameters has been progressively explored. In the following sub-sections, the overall shape and internal medium variations have been introduced one after the other; in order to understand their impact on the muffling behaviour. The concepts developed in the analysis models elaborated in the previous chapters form the basis of the capability building, necessary to achieve optimized designs.

6.1.1 Simple muffling systems with no losses involved

Within axi-symmetric cross section based configurations, the simplest one is the circular muffler, also termed as the simple expansion chamber. The study of this fundamental structure can be used as a benchmark for the complex muffling configurations (Munjal, 2014). The introduction of gradual enhancements, with regards to shape variation, muffling sections have been improved and thus compared. These variations include the simple conical or linear flared mufflers followed by the higher order power functions in terms of the quadratic and the square root based sections. The devices covered in this section have been considered in the absence of the existence of any lossy medium or internal structures to follow the exclusive impact of the structure based variations. Their entire structure essentially consists of an axi-symmetric expansion section appended by like tubes but of smaller cross sectional area; serving as the inlet and outlet tubes. The plane wave is incident through the inlet opening and the terminating end of the outlet tube is maintained such that it allows the downstream propagating plane wave to radiate outwards into the ambience. These structures could act as generic external designs for the initiating of a particular study of reactive type of silencers or mufflers.

The profiles discussed as hereunder have been studied analytically and further detailed upon and validated using the semi-analytical and numerical models. The transmission and absorption characteristics and the pressure, velocity profiles have been presented as an understanding of the muffling characteristics of various structures described as under.

6.1.1.1. Simple Expansion Chamber (SEC)

The most basic cylindrical shell based structure capable of demonstrating muffling effect is the simple expansion chamber, which has been reviewed in depth in the previous chapters. In order to present a visual understanding, Figure 6.1. shows the SEC modelled in COMSOL, while Figure 6.2. shows its meshed model.

The TMM for the expansion chamber as used can be written in a 2×2 matrix as

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos(k_0 l) & iZsin(k_0 l) \\ \frac{i}{Z}\sin(k_0 l) & \cos(k_0 l) \end{bmatrix} \tag{6.1}$$

where, T is the transmission matrix, k_0 is the wave number, l is the length of the expanding section and Z is the characteristic impedance.

The overall matrix, which is a result of the multiplication of the transmission matrix and the matrices of contraction and expansion, viz. A_{cont} and A_{exp}, gives the expected outcome in terms of the transmission loss (TL) and the reflection and absorption coefficients, for the simple cases discussed as hereunder.

$$A_{cont} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{{r_{min}}^2}{R^2} \end{bmatrix}$$

$$A_{exp} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{R^2}{{r_{min}}^2} \end{bmatrix} \tag{6.2}$$

where, r_{min} and R are the maximum possible internal and external cross sectional dimensions of the flare of the chamber.

The plots of their variations against frequency have been obtained semi-analytically are compared with the COMSOL plots. As the presence of losses has been avoided, the numerically obtained plots trace the COMSOL based plots. Figure 6.3 represents the transmission loss characteristics that have been obtained using the FEM solver.

A simple cylindrical system having dimensions of $2\ cm$ radius and $20\ cm$ length, causes an acoustic power loss slightly greater than $6.5\ dB$ in between the inlet and outlet. Fig. 6.4. collectively presents the coefficients. It can be seen that the effect of the expansion within the chamber causes the sound waves to fill in and reflect, accounting for the higher reflection coefficient values. The rest is transmitted which is represented by the green curve. The principle of energy conservation is followed here as the losses are absent and under ideal conditions the absorption curve is a straight line along zero.

The first harmonic peak for the transmission characteristics occurs as a frequency of around $450\ Hz$ while the corresponding trough occurs at nearly double its value of $900\ Hz$. The pressure and local instantaneous velocity variations within the tube at these two frequencies have been captured and presented through the Figures 6.5, 6.6 and Figures 6.7, 6.8 respectively. In the acoustic pressure plots it can be noted that the total acoustic pressure field propagating within the system initiating from the inlet end is a combination of the scattered and the reflected component. There is a strong rarefaction towards the termination of the chamber as the wave traverses at the two selected frequencies. Similarly in case of the velocity plots, when the TL reaches its first peak, the velocity progressively weakens as the wave enters the muffling section. The plots show strong influence of the sudden area changes at the sudden expansion and contraction ends of the SEC.

The Sound Pressure Level (SPL) in dB has been plotted for the two frequencies (Figures 6.9 and 6.10) to understand the drop in the levels due to the presence of and expansion section within a tube configuration. The SEC in the absence of losses shows a loss in the level by 10 dB at frequency corresponding to peak TL. In the TL dip region, there is no variation in the SPL between the inlet and the outlet sections as expected.

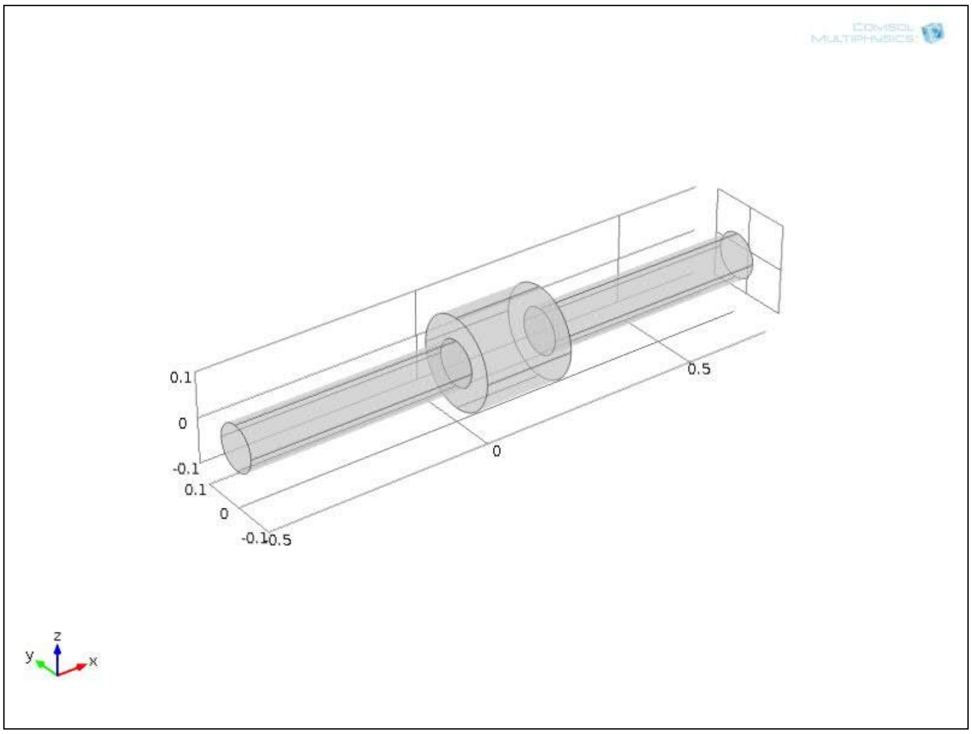

Figure 6.1: Simple Expansion Chamber model

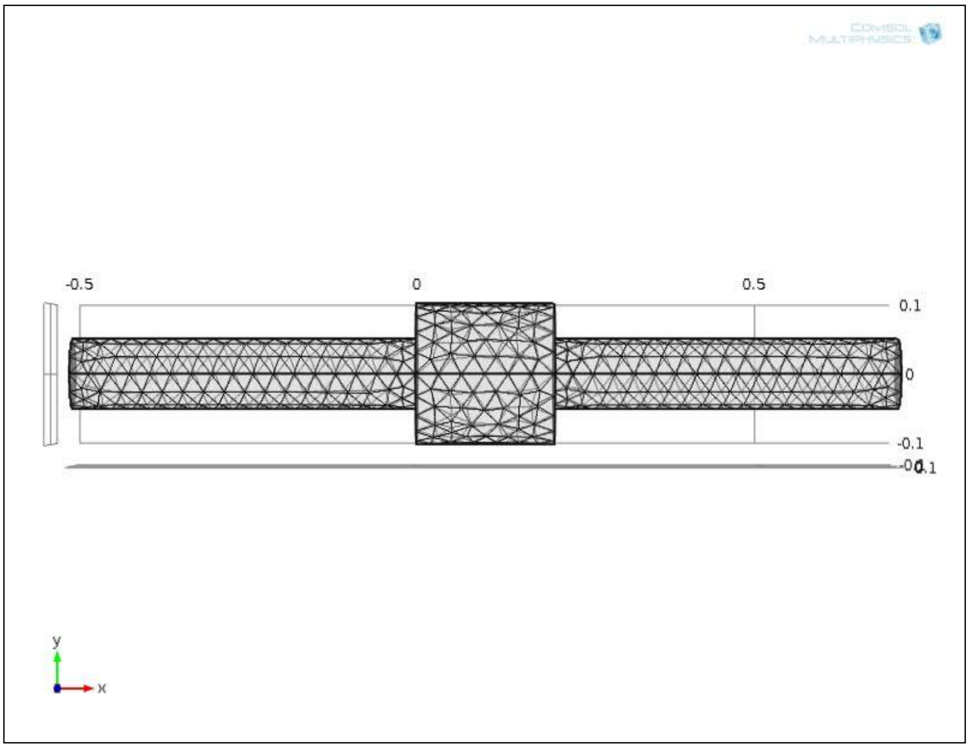

Figure 6.2: Mesh model of Simple Expansion Chamber

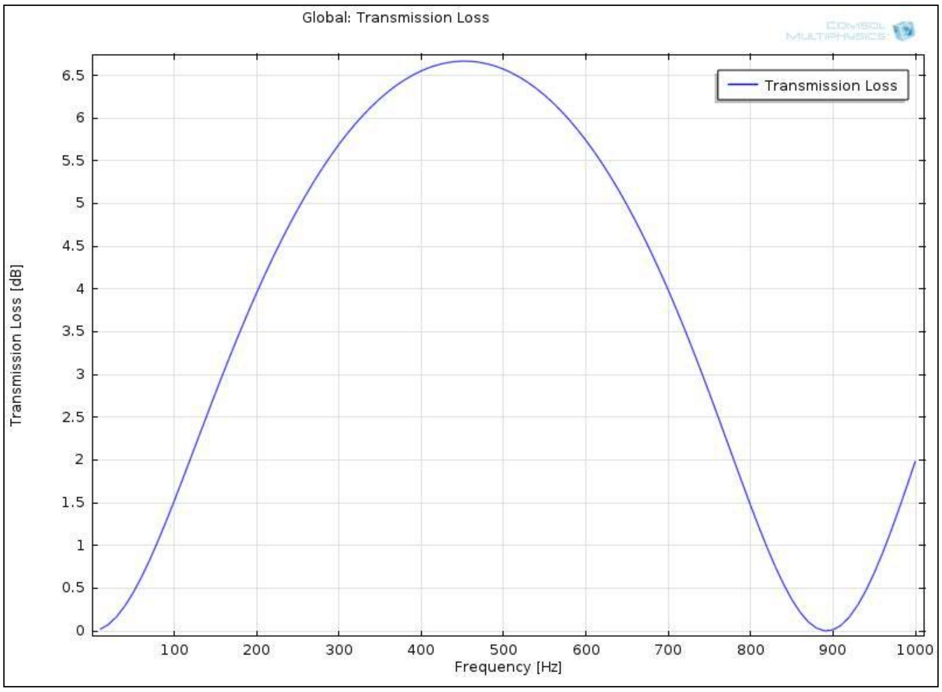

Figure 6.3: Transmission Loss characteristics (dB) against frequency (Hz) for Simple Expansion Chamber

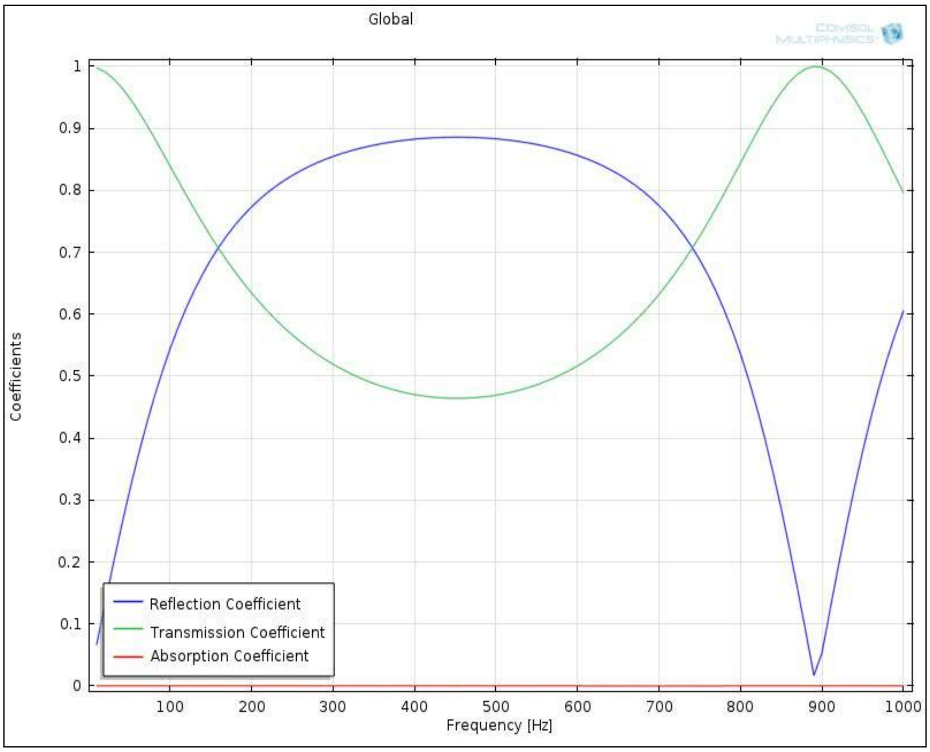

Figure 6.4: Reflection, Transmission and Absorption Coefficient plots against frequency (Hz) for Simple Expansion Chamber

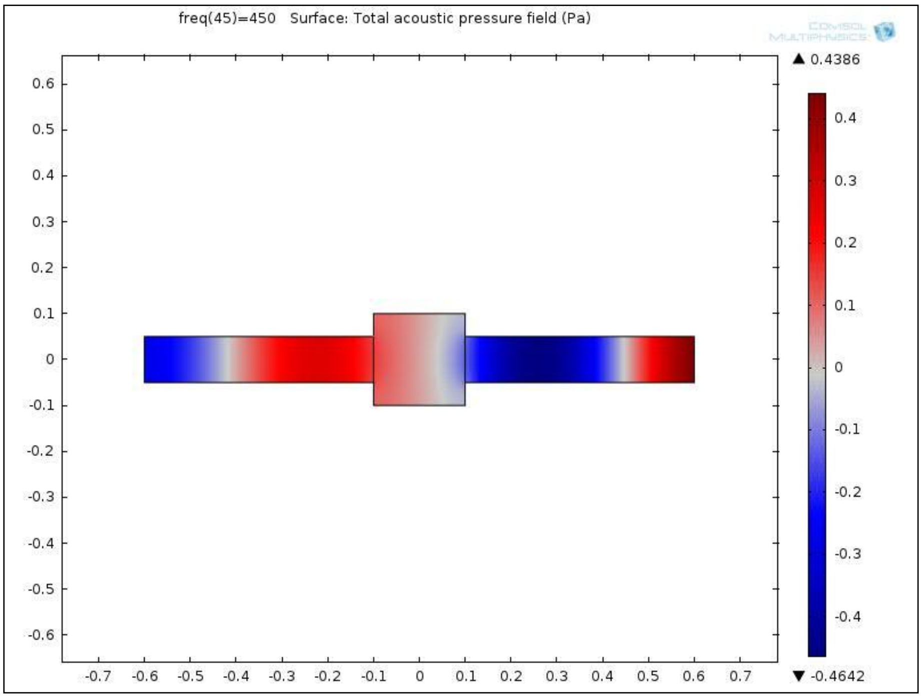

Figure 6.5: Total acoustic pressure field (Pa) within the Simple Expansion Chamber at 450 Hz frequency

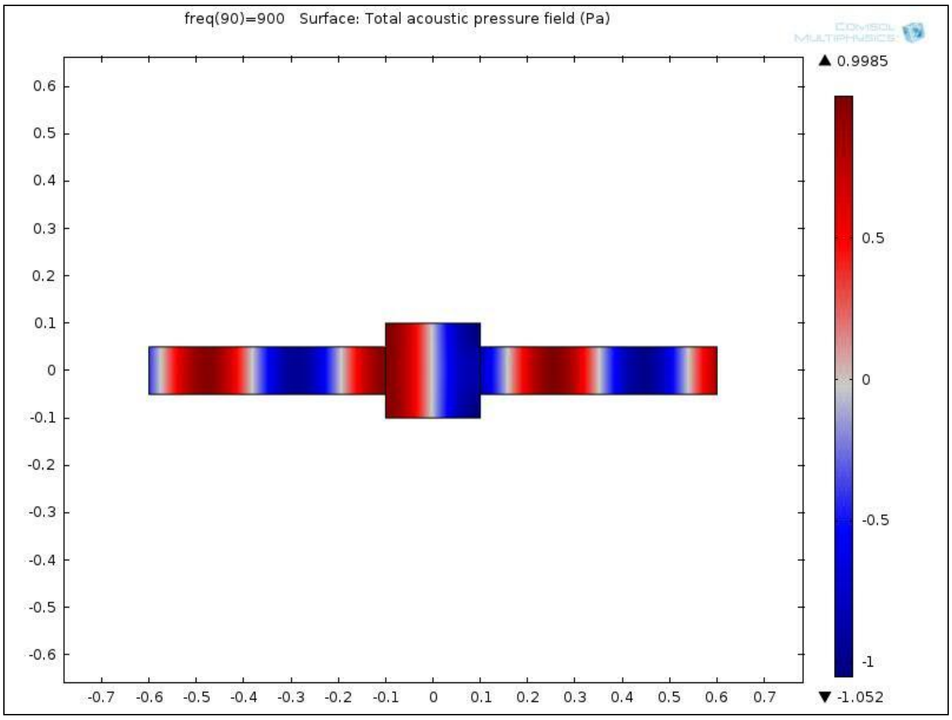

Figure 6.6: Total acoustic pressure field (Pa) within the Simple Expansion Chamber at 900 Hz frequency

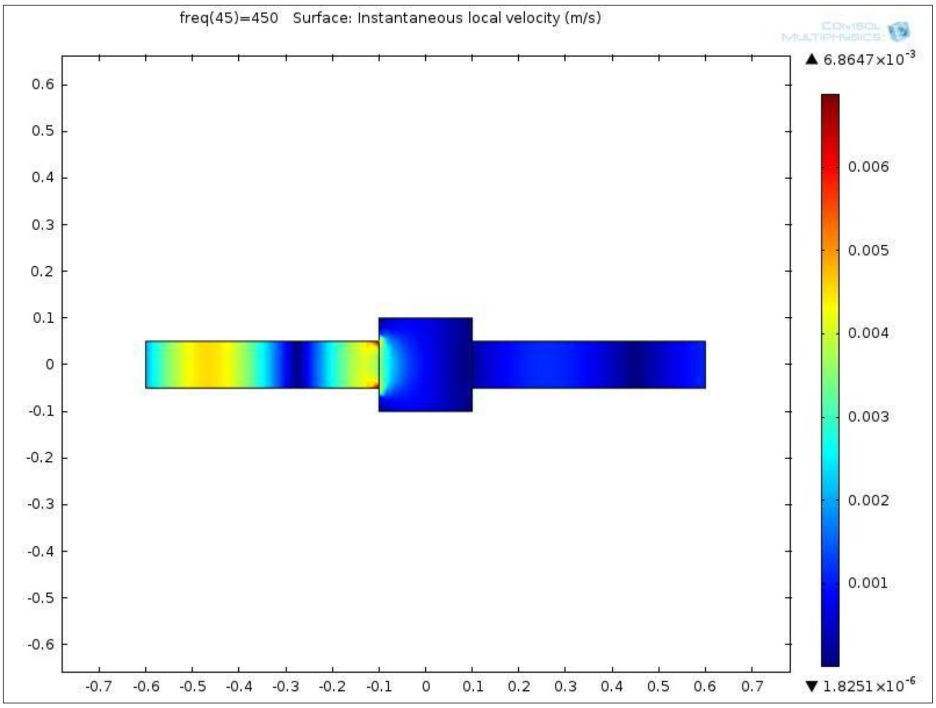

Figure 6.7: Instantaneous Local Velocity (m/s) within the Simple Expansion Chamber at 450 Hz frequency

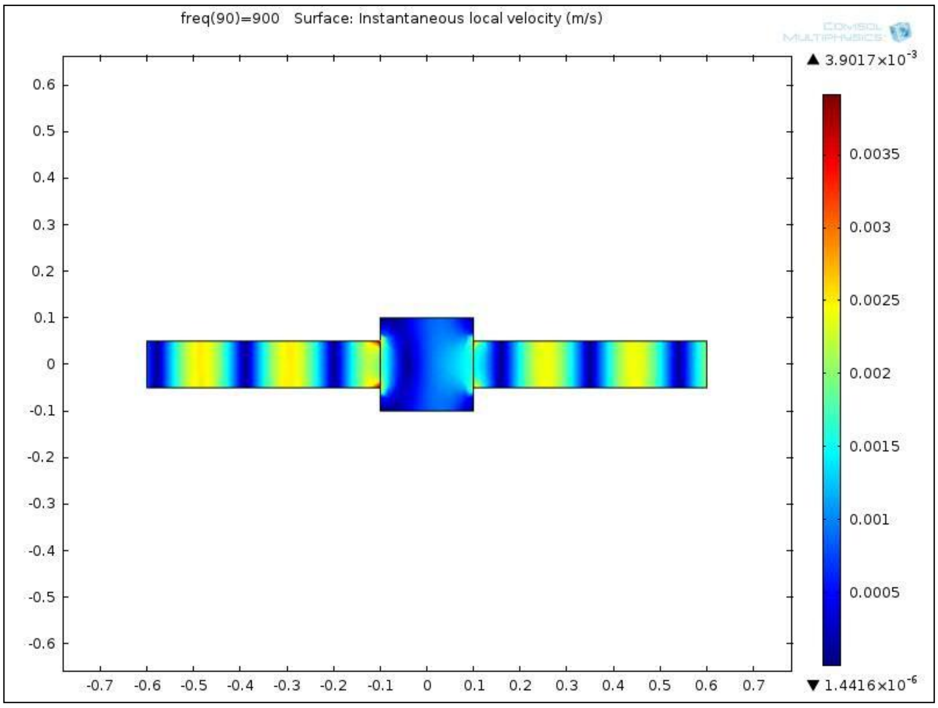

Figure 6.8: Instantaneous Local Velocity (m/s) within the Simple Expansion Chamber at 900 Hz frequency

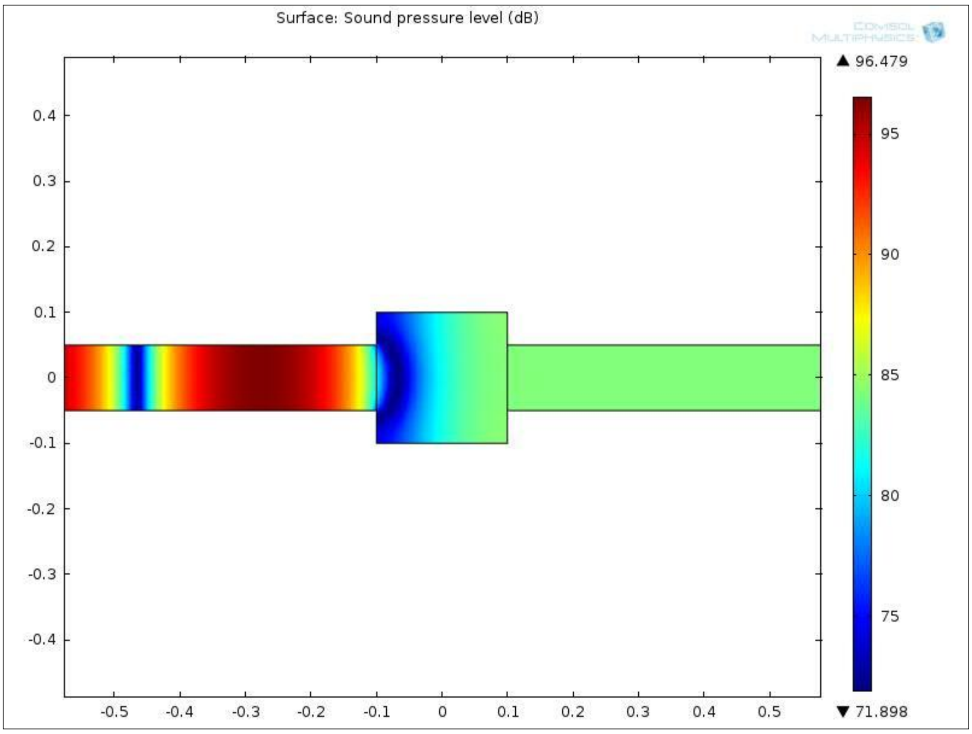

Figure 6.9: Sound Pressure Level (dB) within the Simple Expansion Chamber at 450 Hz frequency

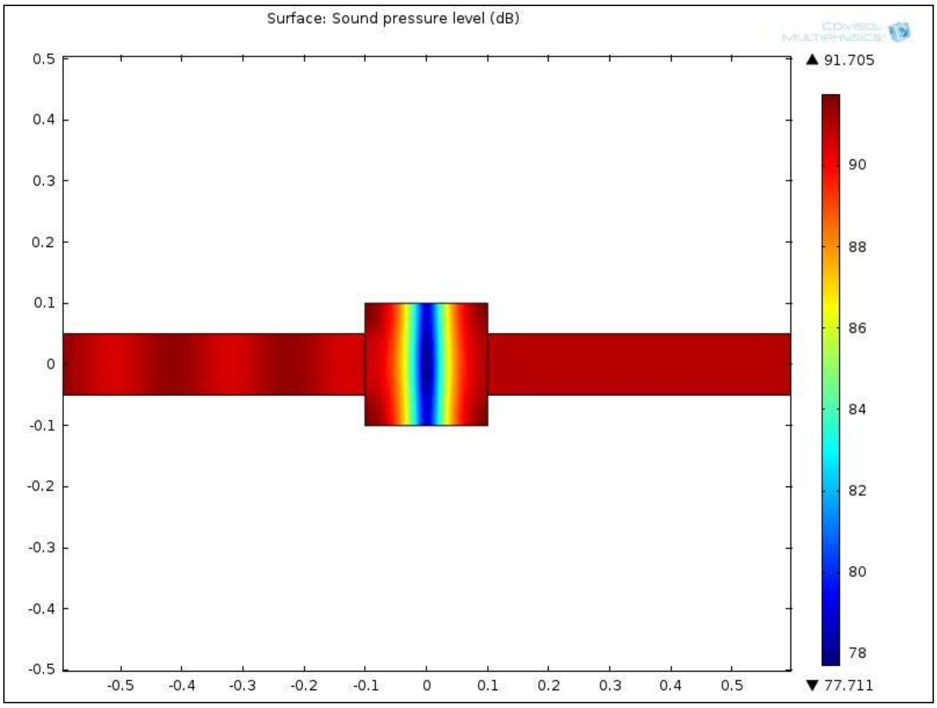

Figure 6.10: Sound Pressure Level (dB) within the Simple Expansion Chamber at 900 Hz frequency

6.1.1.2. Simple Conical Chamber (SCC)

A slight variation to the SEC is the SCC where the cylindrical chamber has been replaced by a tapering section. The conical section occupies one-third the volume of the SEC chamber and thus can be utilized in situations having tighter, much narrower, space constraints. The FEM model illustrated in the Figure 6.11 has a slow increasing taper at the entrance and a sudden contraction at its termination, similar to SEC. Figure 6.12 shows its meshed model.

The semi-analytical model has been developed based on the TMM that can be obtained by general algebraic simplification as,

$$[T] = \begin{bmatrix} \frac{d_l}{d_0}\cos(k_0 l) - \frac{m}{k_0 d_0}\sin\,(k_0 l) & iZ\frac{d_0}{d_l}sin(k_0 l) \\ \left\{\frac{i}{Z}\frac{d_l}{d_0}\left(1+\frac{m^2}{k_0^2 d_0 d_l}\right)\sin(k_0 l) - \frac{im}{k_0 d_0 Z}\frac{d_l}{d_0}\left(1-\frac{d_0}{d_l}\right)\cos(k_0 l)\right\} & \left\{\frac{m}{k_0 d_l}\sin(k_0 l) + \frac{d_0}{d_l}\cos(k_0 l)\right\} \end{bmatrix} \quad (6.3)$$

where, d_0 and d_l are the equivalent diameters of the conical chamber at the entrance and exit of its tapering section or the junctions with the inlet and the outlet tube and $m = \frac{(d_l - d_0)}{l}$ is the slope of the cone representing the muffling section.

The TL and coefficient plots in Figure 6.13 and Figure 6.14 have been obtained as before. It can be observed that that in comparison to the TL as obtained by the SEC, the SCC provides slightly lesser peak value of around $3.4\ dB$ occurring at a much lower frequency of around $350\ Hz$. In specifically the low frequency ranges, the loss across the section as provided by the configuration as depicted by the conical model presented (See Fig. 6.11) is congruent to the TL that would be obtained in the case of an inverse conical section; such that the muffling section has a sudden expansion and a gradually tapering termination. In the absence of a lossy medium the muffling effect provided by both these configurations is the same.

The coefficients plotted suggest that in comparison to the SEC, there is higher amount of transmission of the acoustic wave and thus consequently lower reflection within the chamber. Due to the absence of losses, there is expectedly no visible absorption present.

The frequencies of the first peak and trough as obtained in the TL plots have been probed for the 2D axially cut planes for observing the acoustic pressure and the local instantaneous velocity. Fig. 6.15-6.18 show the variations accordingly.

The pressure plots depict the gradual propagation of the total acoustic pressure field with the contractions and rarefactions getting narrower as the frequencies rise. The velocity plots show that at both frequencies, the travelling wave slows down as it progresses through the muffler and dips to a minimum at its terminating end. The lowest speed zones are developed near the region where the taper meets the sharp contraction.

The SPL in dB has also been considered for the two frequencies. They are represented in the Figure 6.19 and Figure 6.20. While there was a $10\ dB$ drop for the SEC model, the SCC shows a much lower drop in the levels.

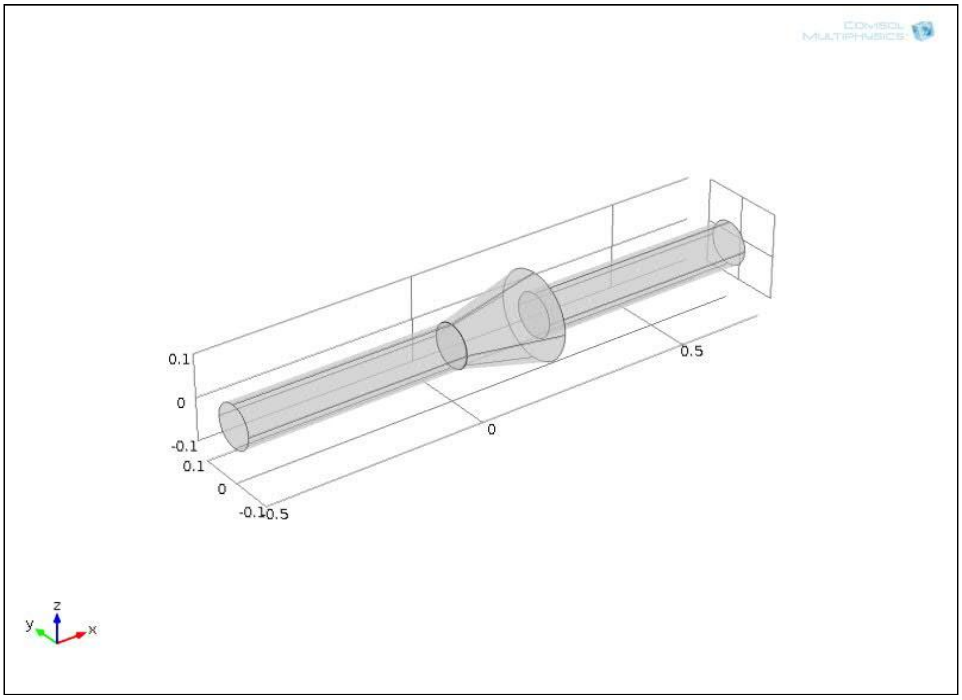

Figure 6.11: Simple Conical Chamber model

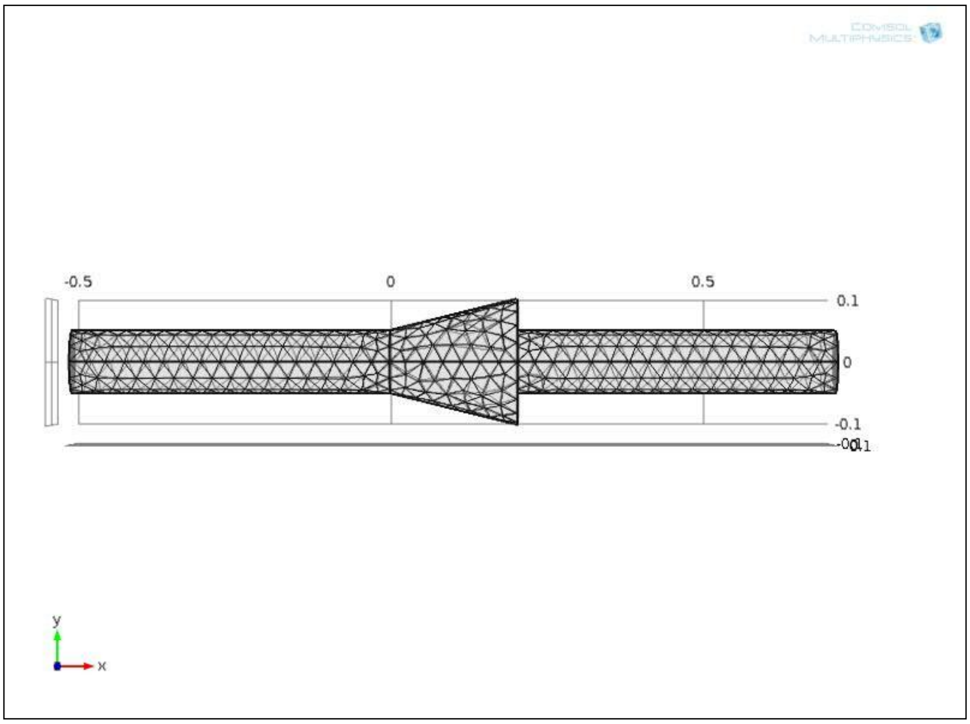

Figure 6.12: Mesh model of Simple Conical Chamber

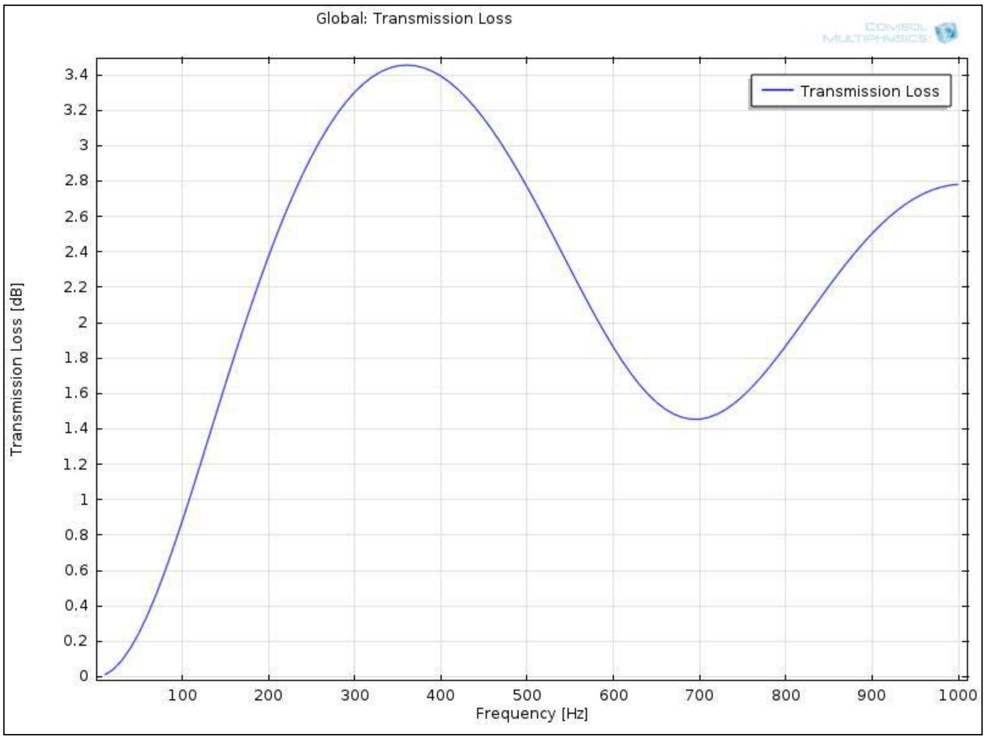

Figure 6.13: Transmission Loss characteristics (dB) against frequency (Hz) for Simple Conical Chamber

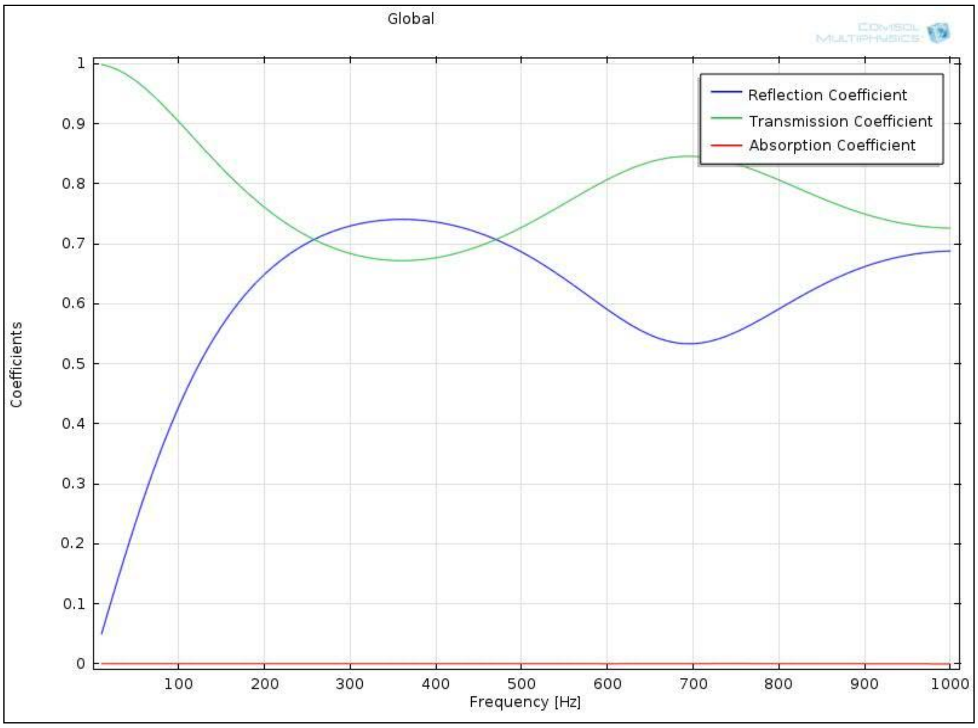

Fig 6.14. Reflection, Transmission and Absorption Coefficient plots against frequency (Hz) for Simple Conical Chamber

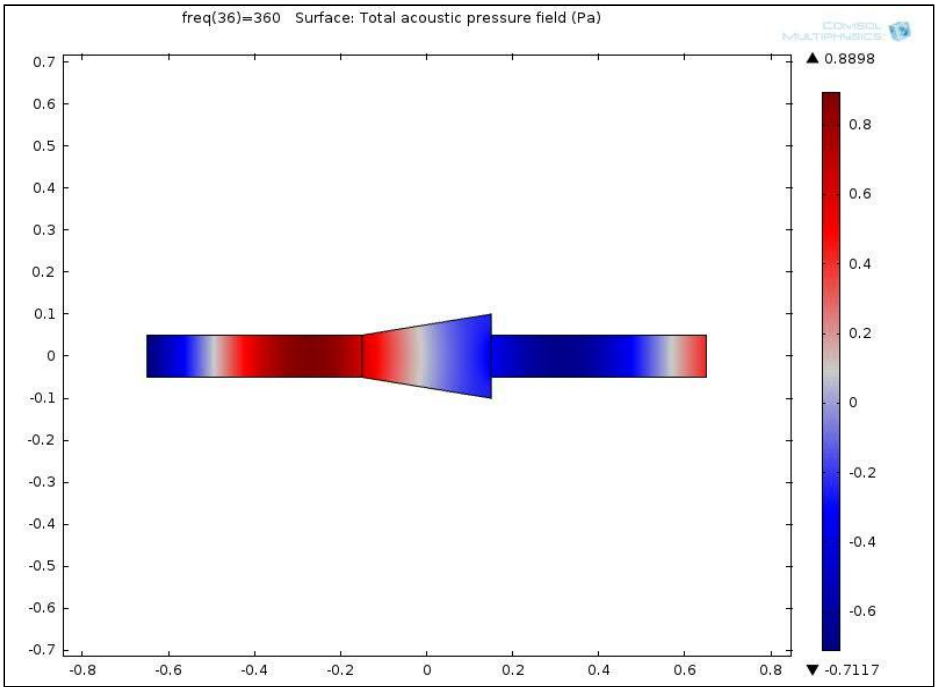

Figure 6.15: Total acoustic pressure field (Pa) within the Simple Conical Chamber at 360 Hz frequency

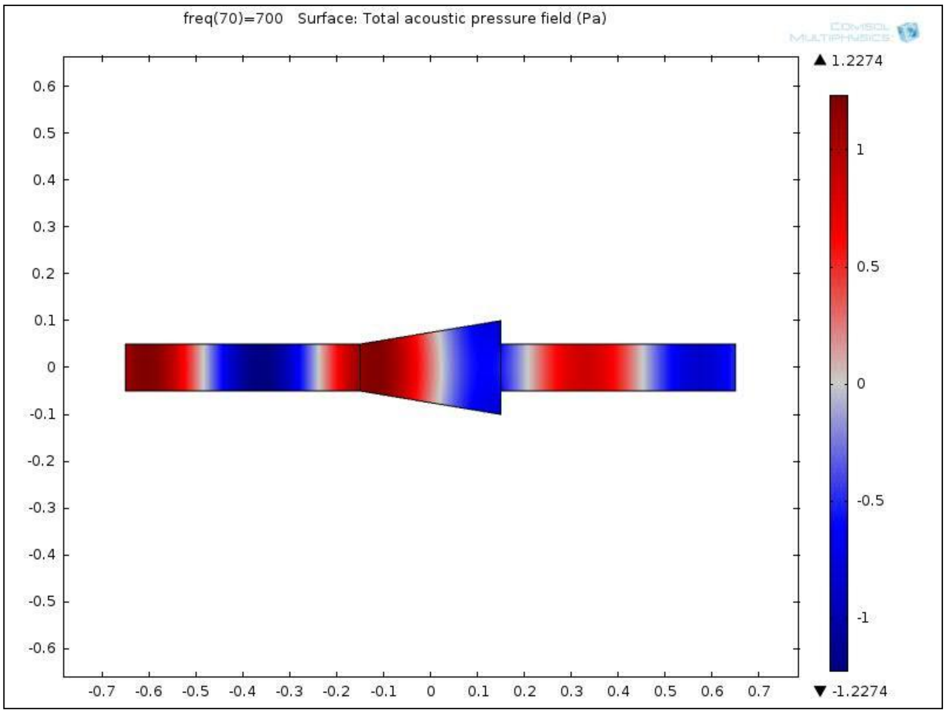

Figure 6.16: Total acoustic pressure field (Pa) within the Simple Conical Chamber at 700 Hz frequency

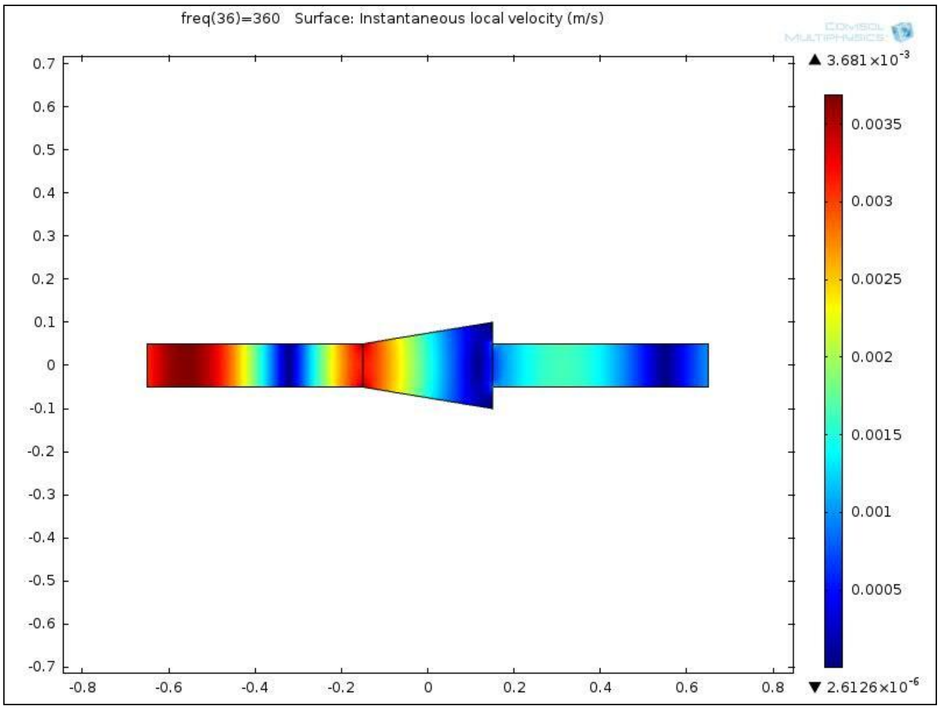

Figure 6.17: Instantaneous Local Velocity (m/s) within the Simple Conical Chamber at 360 Hz frequency

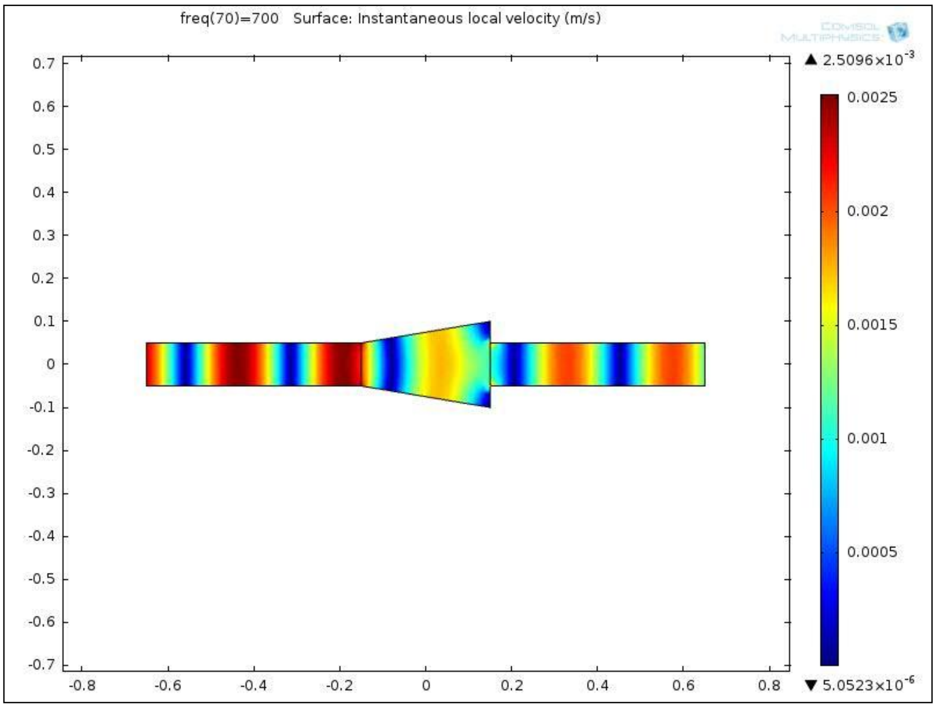

Figure 6.18: Instantaneous Local Velocity (m/s) within the Simple Conical Chamber at 700 Hz frequency

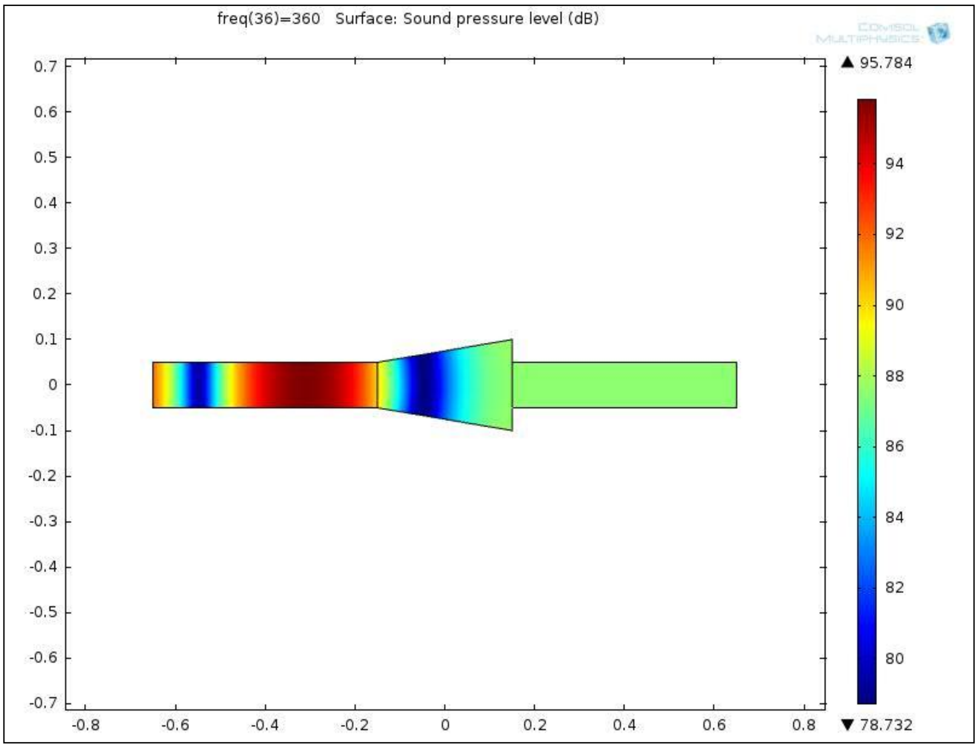

Figure 6.19: Sound Pressure Level (dB) within the Simple Conical Chamber at 360 Hz frequency

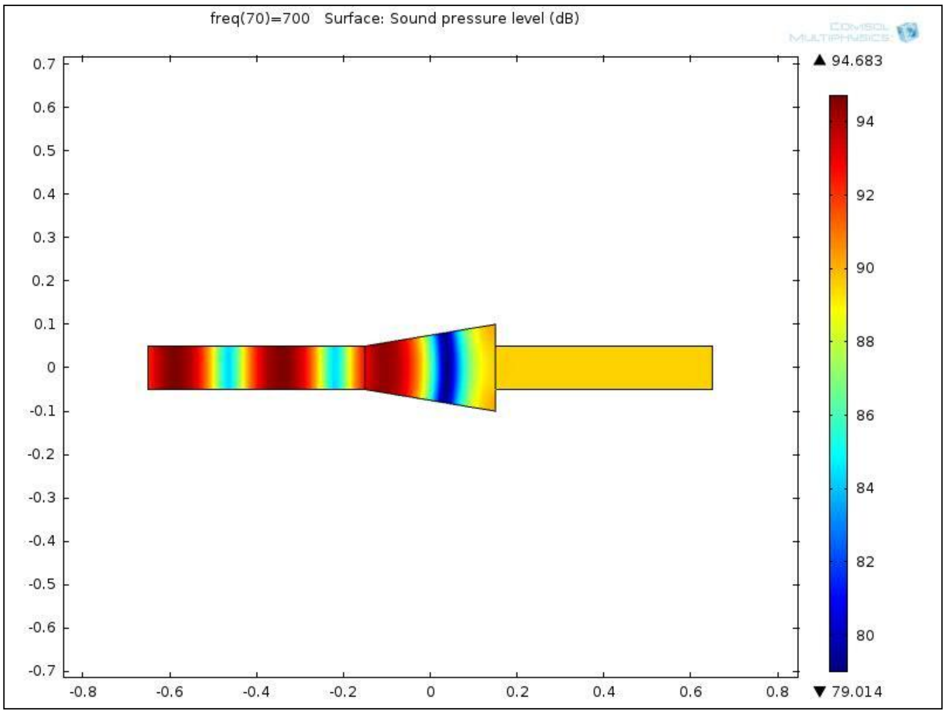

Figure 6.20: Sound Pressure Level (dB) within the Simple Conical Chamber at 700 Hz frequency

6.1.1.3. Simple Quadratic Chamber

While the conical chamber has a linear tapering profile, a variation to this is in the form of a quadratic curve based muffling unit. This is depicted in the Figure 6.21 modelled using the FEM solver. Figure 6.22 gives a zoomed view of the meshed model. The mesh towards the edges, surface changes and connecting surfaces is progressively set finer in comparison to the inner surfaces.

Being a power law based profile, it is considered to be a stable performer over the linearly tapered wave guide, which will be evident in the later section. The TL and the coefficients have been plotted for the model in Figure 6.23 and Figure 6.24. In comparison to the conical muffling section, TL and reflection coefficient is lower in its peak amplitude, which occurs at a much later frequency of about 550 Hz.

The 2D profiles of pressure and instantaneous velocity have been captured in Figure 6.25 and Figure 6.26 for the frequency of $570\,Hz$. These figures show the slow consistent propagation within the chamber. Similarly the variation of SPL shows slight dip in the level as per Figure 6.27.

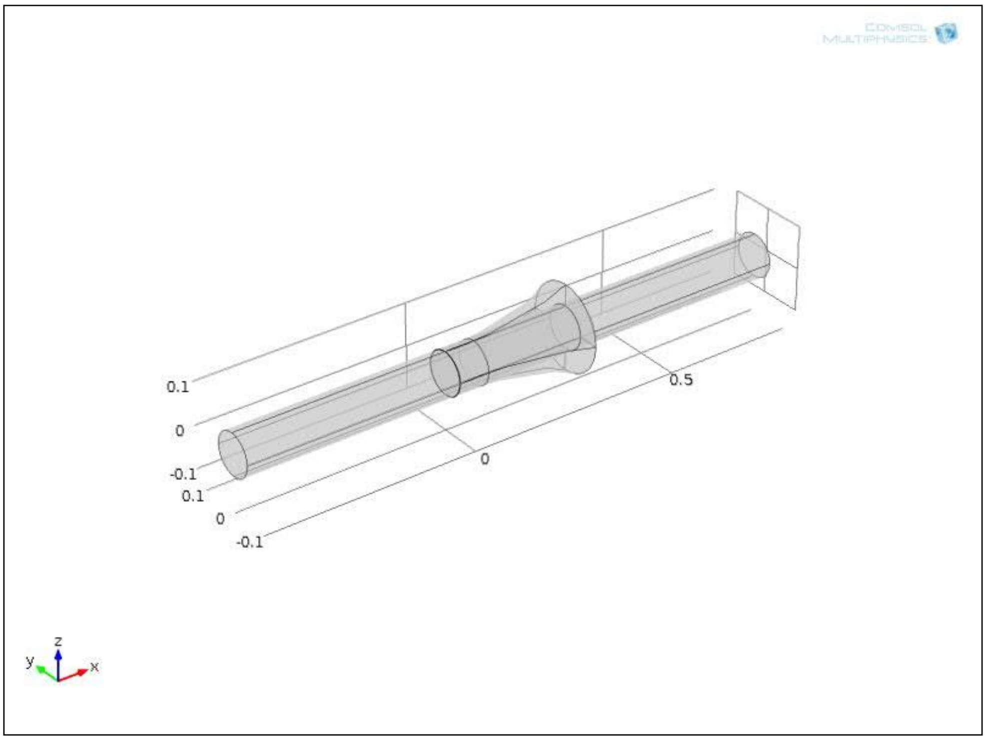

Figure 6.21: Simple Quadratic Chamber model

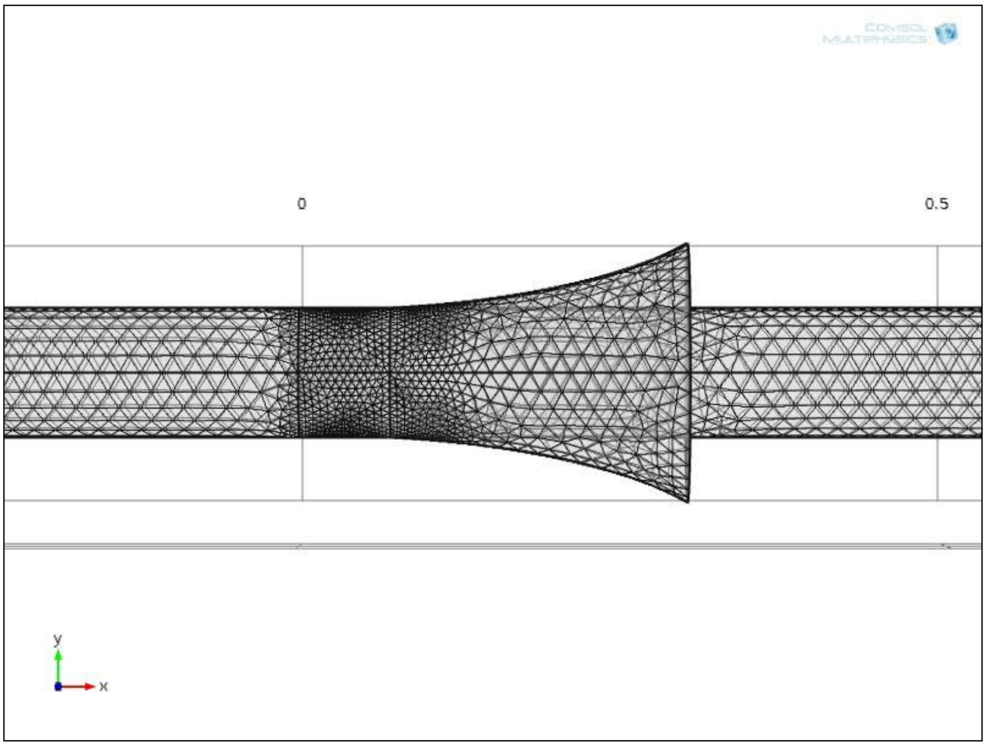

Figure 6.22: Mesh model of Simple Quadratic Chamber

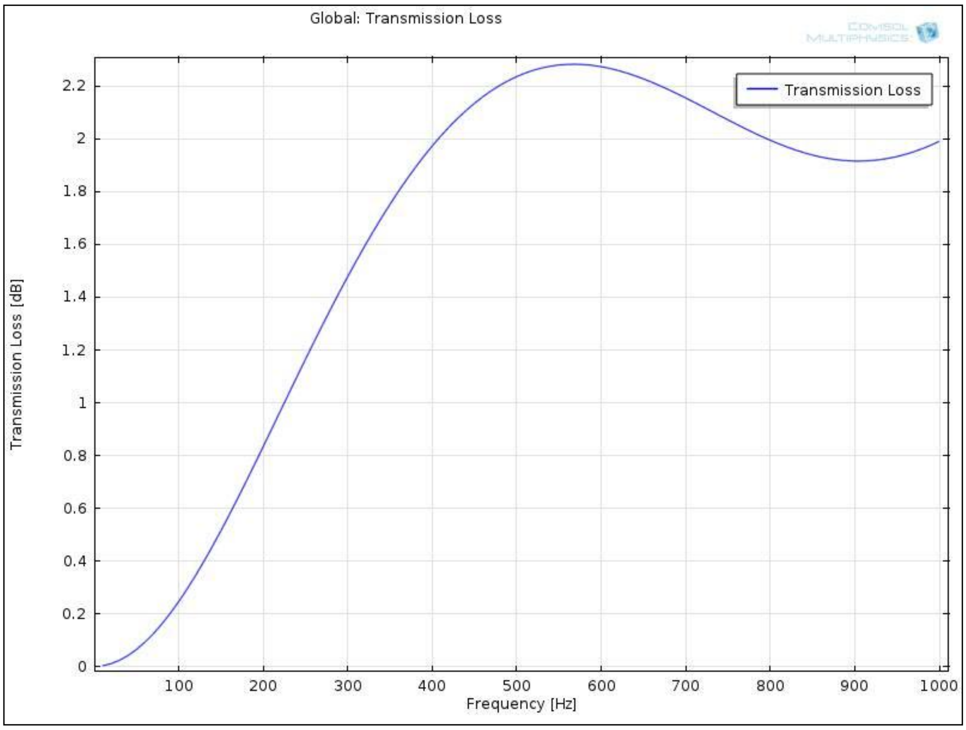

Figure 6.23: Transmission Loss characteristics (dB) against frequency (Hz) for Simple Quadratic Chamber

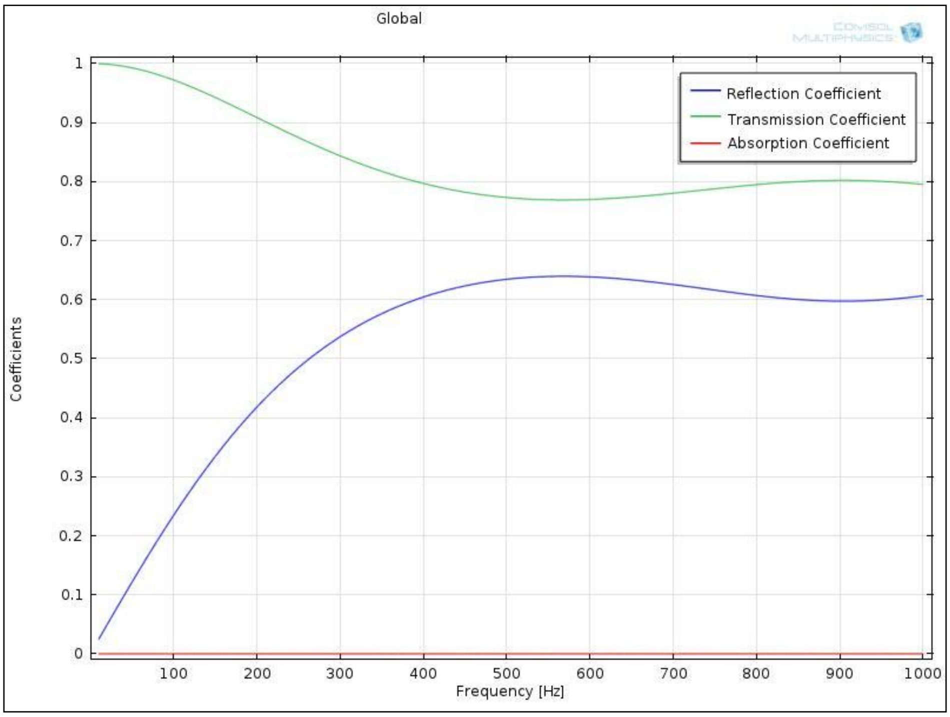

Figure 6.24: Reflection, Transmission and Absorption Coefficient plots against frequency (Hz) for Simple Quadratic Chamber

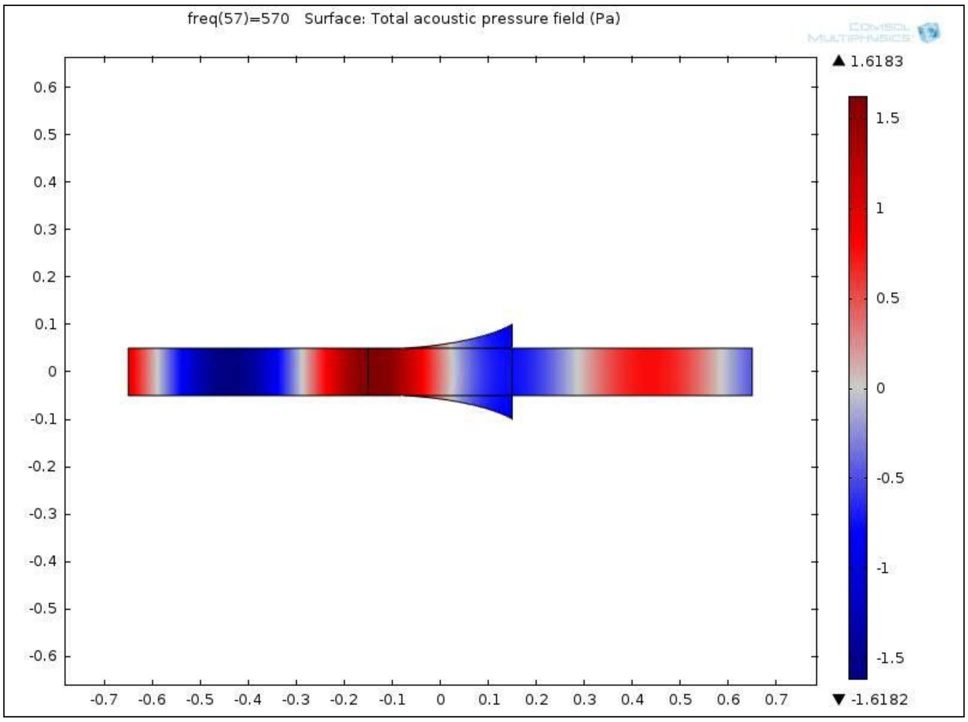

Figure 6.25: Total acoustic pressure field (Pa) within the Simple Quadratic Chamber at 570 Hz frequency

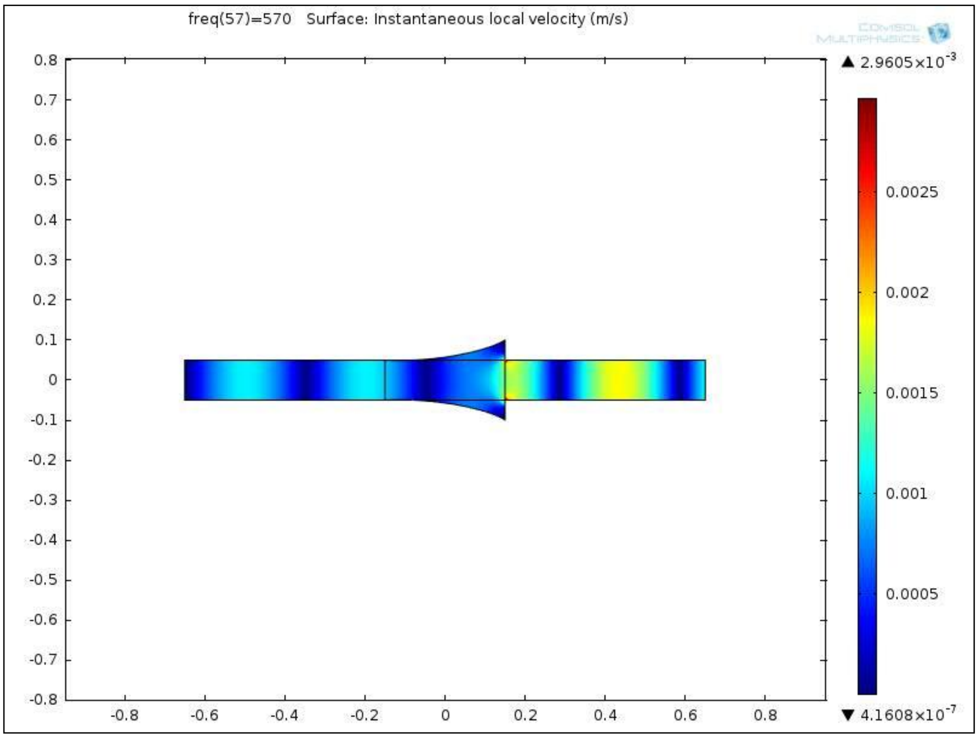

Figure 6.26: Instantaneous Local Velocity (m/s) within the Simple Quadratic Chamber at 570 Hz frequency

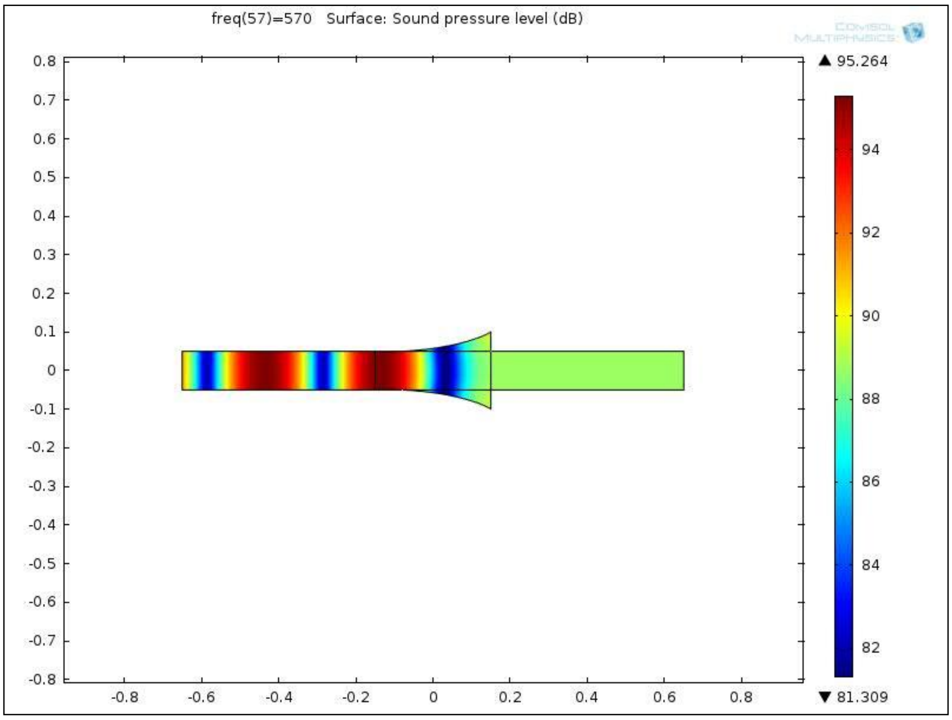

Figure 6.27: Sound Pressure Level (dB) within the Simple Quadratic Chamber at 570 Hz frequency

6.1.1.4. Simple Quadratic Root Chamber

As the quadratic curve varied in close proximity to the tube configuration itself, it could have been responsible to the visibly low performance. Thus, an opposite variation that followed this was in the form of the quadratic square root profile based expansion chamber. The internal bulge could be treated as a comparable system to the SEC, the difference being its graded waveguide nature. The model is as shown in Figure 6.28 and in the meshed format is as in Figure 6.29. As this model is nearly of the same capacity as that of the SEC, the muffling effect is expected to be tantamount. TL plot as in Figure 6.30 is suggestive of this. The TL peak occurs at around $6.5\ dB$, which is the same as that for the SEC. However, there is slight advantage that this peak occurs at a much lower frequency, nearly $100\ Hz$ lower. In the absence of a lossy medium, reflection coefficient is much higher than that of the SEC and complementing transmission coefficient is comparably lower, as per Figure 6.31.

The frequency of the TL peak of $350\ Hz$ and the trough of nearly $700\ Hz$ are the ones for which the acoustic pressure and velocity plots have been extracted as in Figures 6.32-6.35. From these plots it is clear that the design is quite favourable for demonstrating the muffling behaviour. The pressure and velocity variations are smooth and uniform throughout the chamber as also the system as a whole. The SPL shows a sufficient fall of $5\ dB$ across the chamber for both the frequencies (See Figure 6.36 and Figure 6.37). The plane waves traverse evenly within the system.

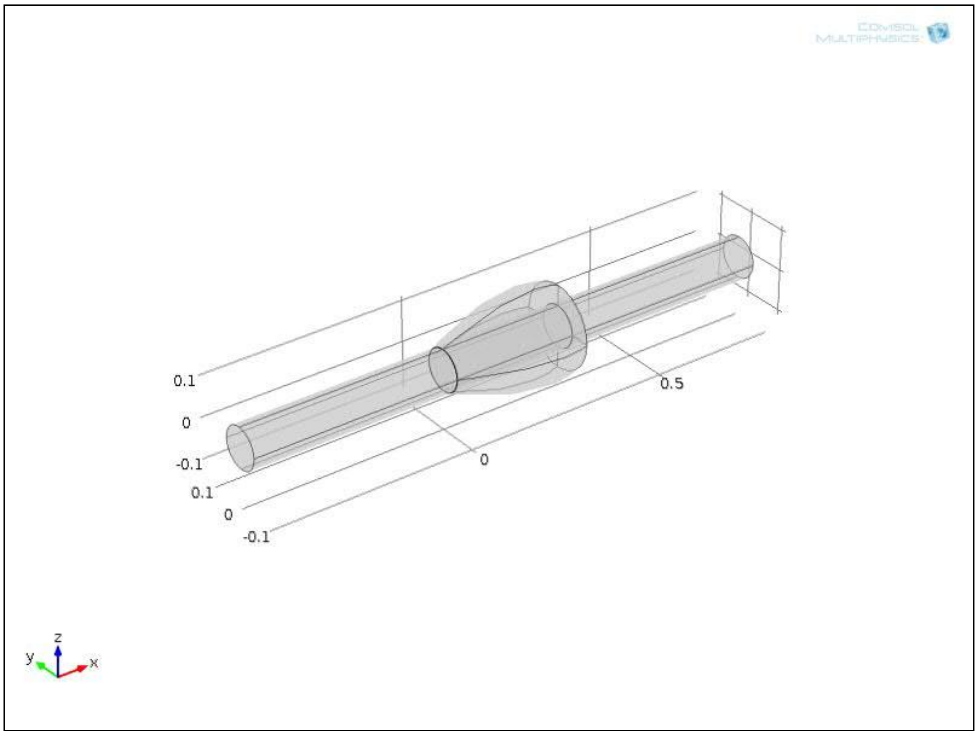

Figure 6.28: Simple Quadratic Root Chamber model

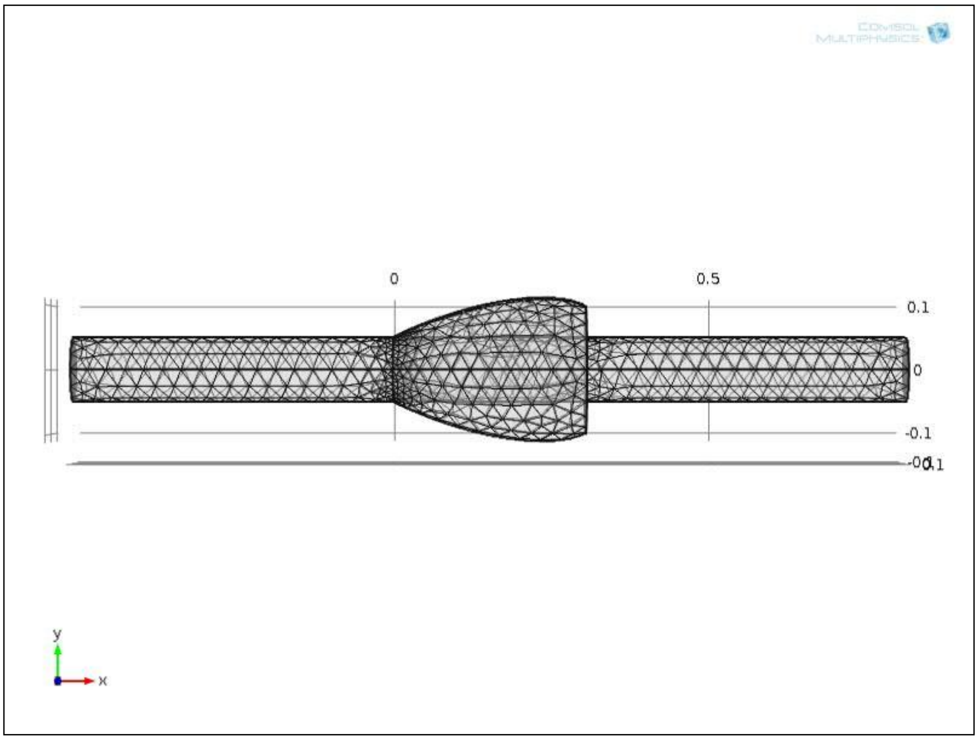

Figure 6.29: Mesh model of Simple Quadratic Root Chamber

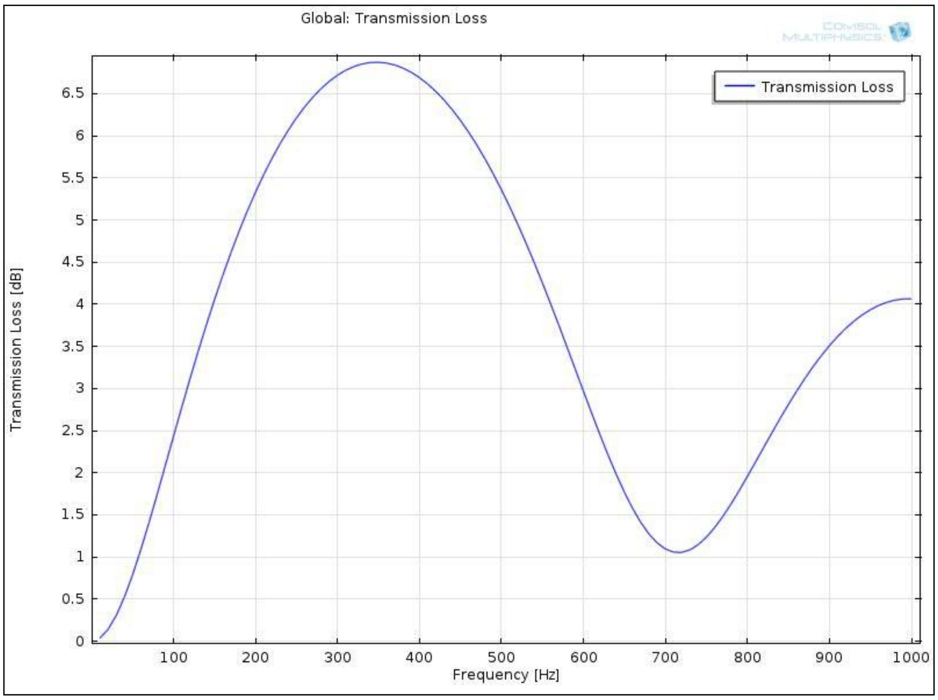

Figure 6.30: Transmission Loss characteristics (dB) against frequency (Hz) for Simple Quadratic Root Chamber

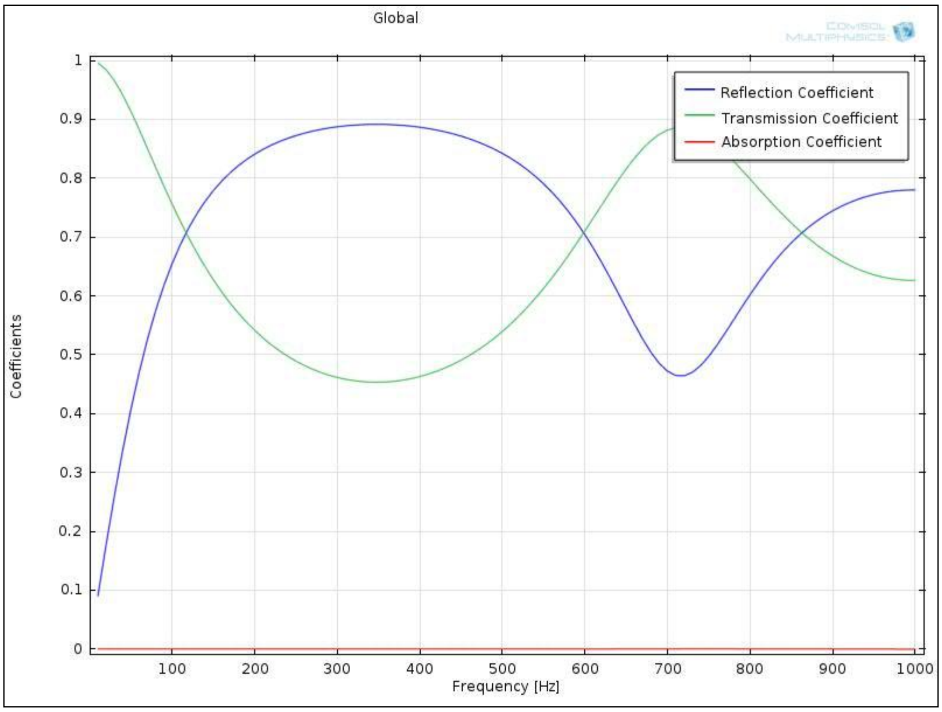

Figure 6.31: Reflection, Transmission and Absorption Coefficient plots against frequency (Hz) for Simple Quadratic Root Chamber

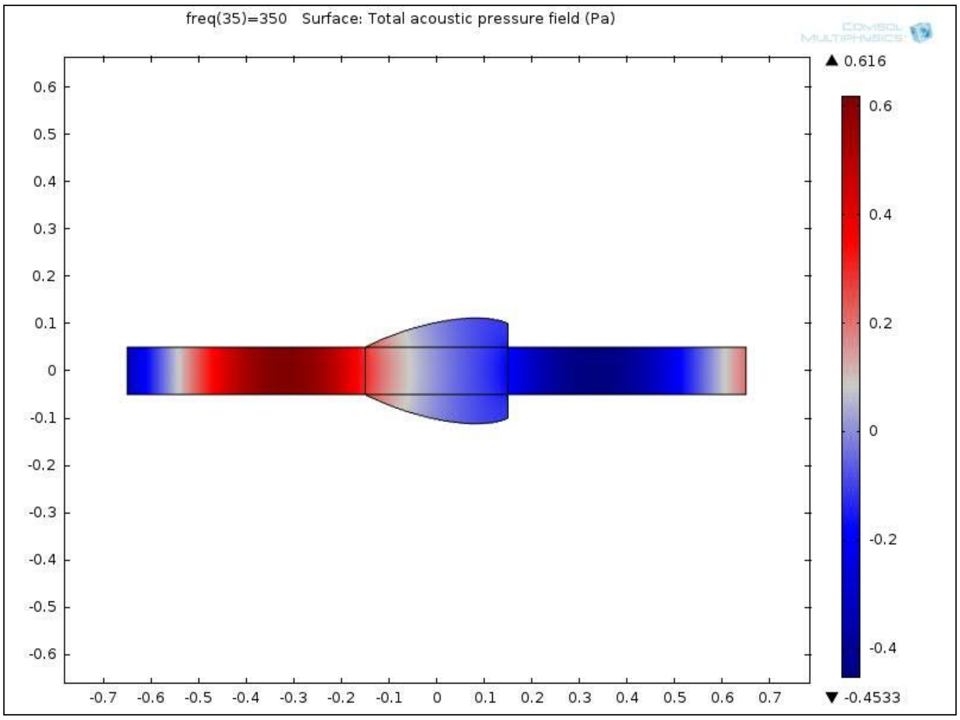

Figure 6.32: Total acoustic pressure field (Pa) within the Simple Quadratic Root Chamber for 350 Hz frequency

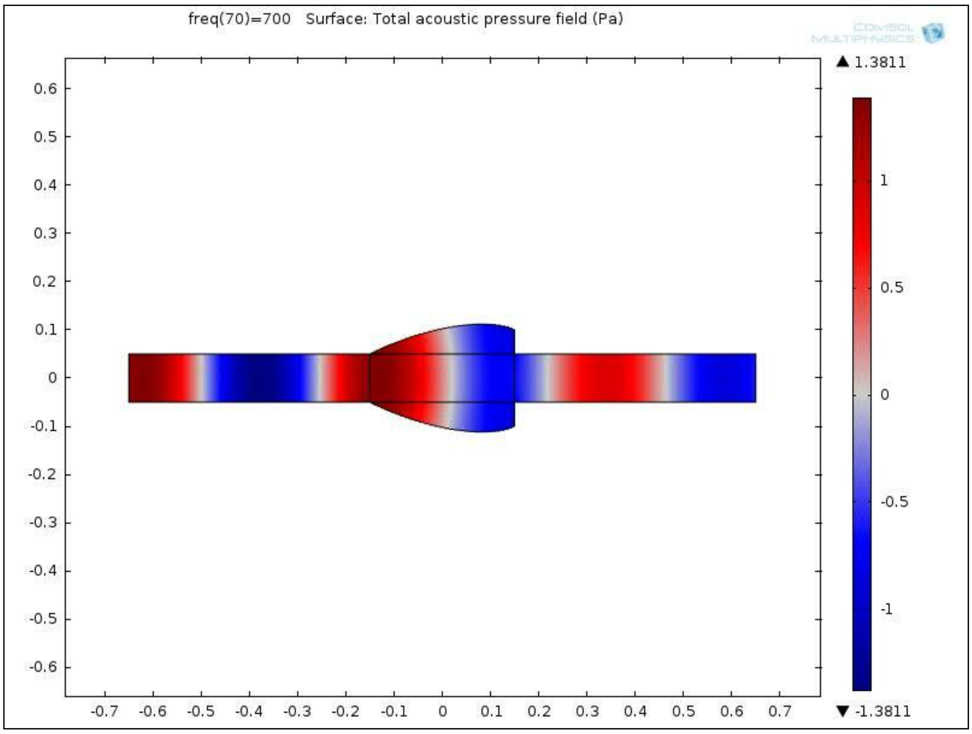

Figure 6.33: Total acoustic pressure field (Pa) within the Simple Quadratic Root Chamber at 700 Hz frequency

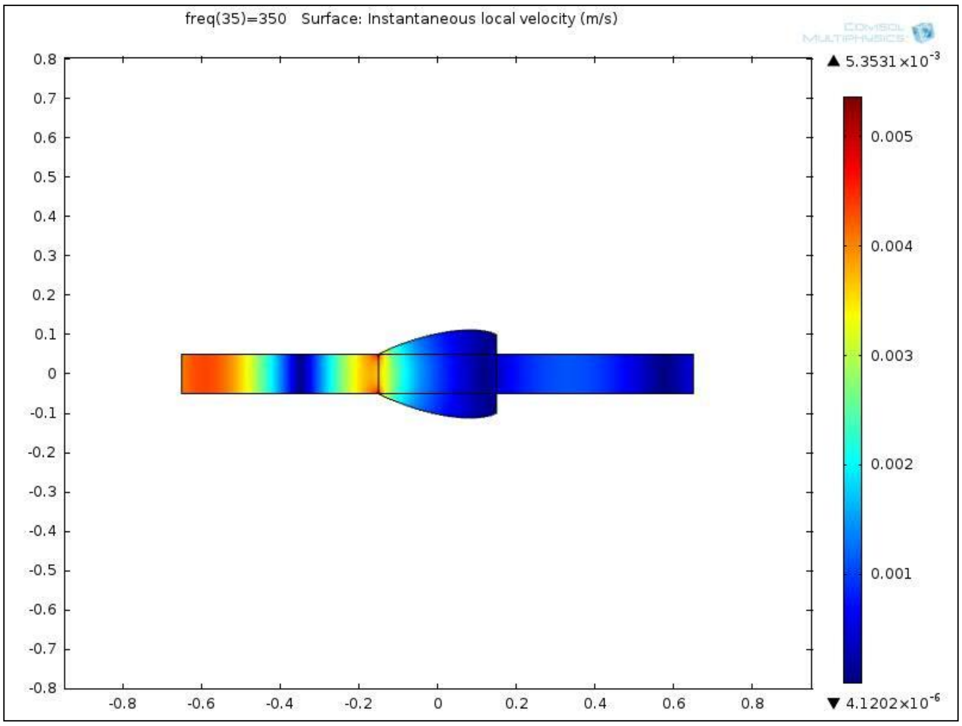

Figure 6.34: Instantaneous Local Velocity (m/s) within the Simple Quadratic Root Chamber at 350 Hz frequency

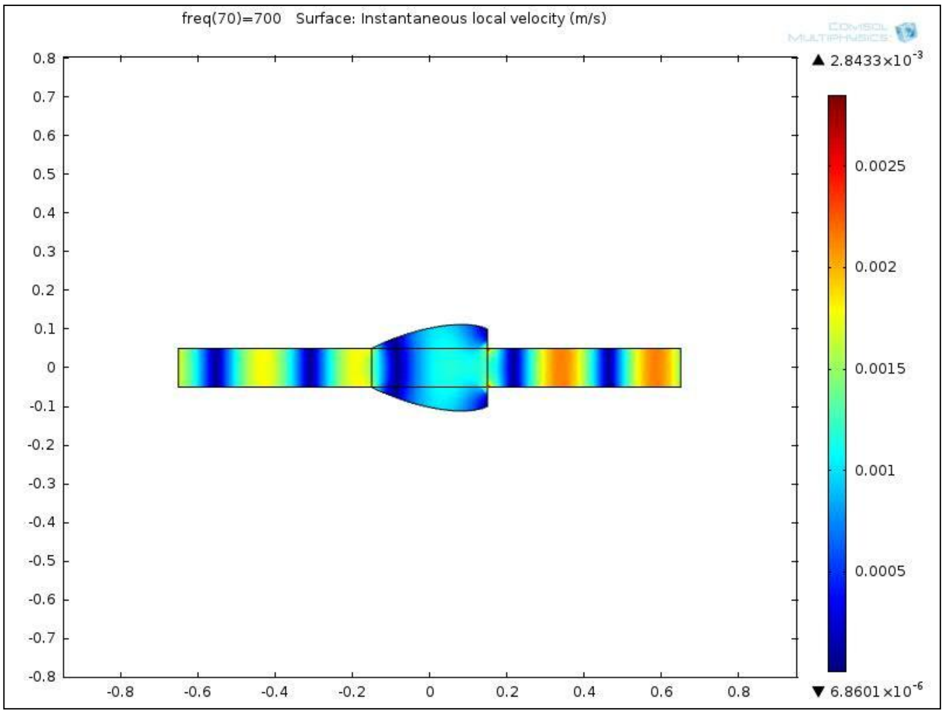

Figure 6.35: Instantaneous Local Velocity (m/s) within the Simple Quadratic Root Chamber at 700 Hz frequency

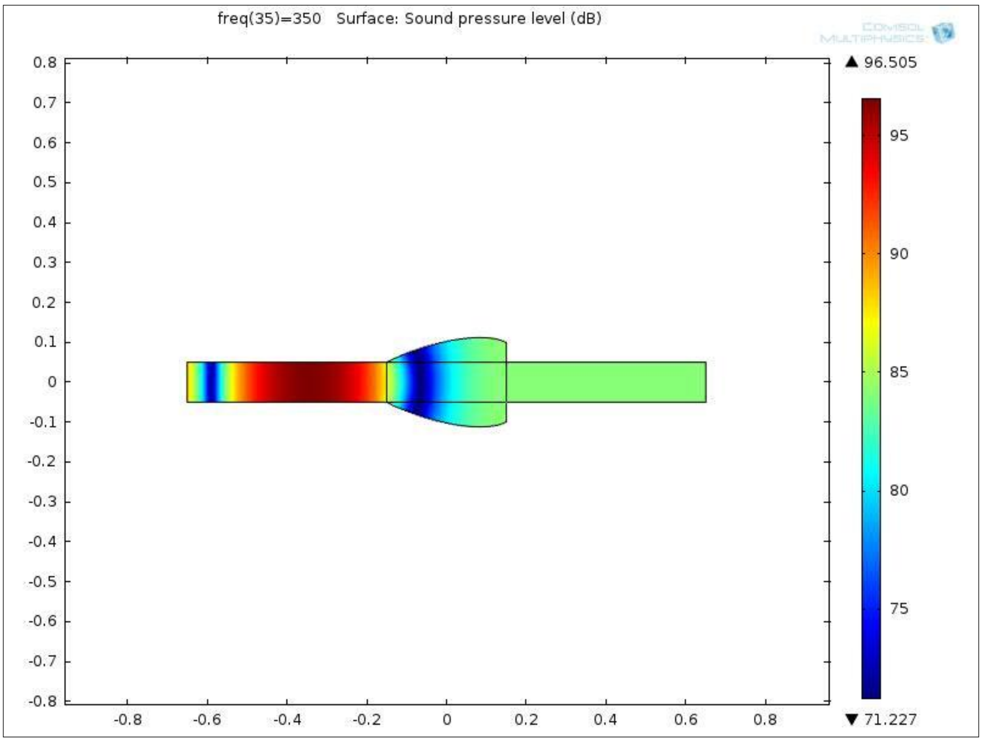

Figure 6.36: Sound Pressure Level (dB) within the Simple Quadratic Root Chamber at 350 Hz frequency

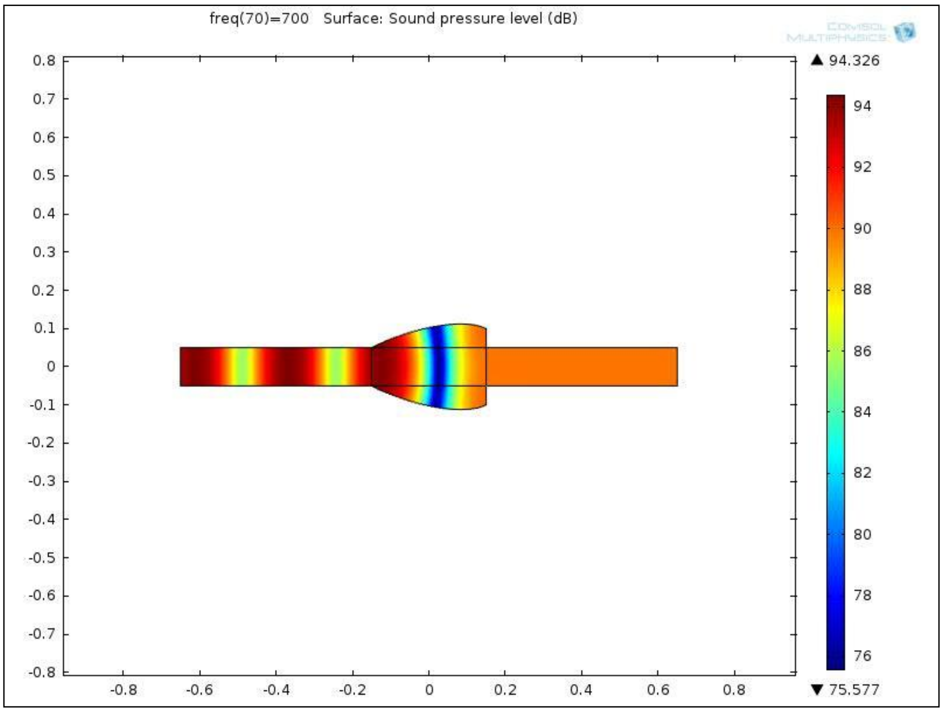

Figure 6.37: Sound Pressure Level (dB) within the Simple Quadratic Root Chamber at 700 Hz frequency

6.1.2. Muffling systems with introduction of internal structures in absence of losses

Simulations have been carried out for a particular set of parameters for the model in Figure 6.38, $L = 0.1$ m, assuming $R_0 = 2 \times r_0$ as a simple relation between the tube and chamber radii, $a = 0.001$ m as the slit half width defining the gap between two adjacent rings, a low frequency range of 10-4k Hz and maintaining the medium dependent parameters based on air at STP (Standard Temperature and Pressure) conditions.

The comparisons of the numerical results for reflection and transmission coefficients with analytical based WKB approximation predictions are shown in Figure 6.39 and Figure 6.40. The WKB based zone of validity has been depicted by the shaded area. A good agreement is demonstrated in the range of frequencies where analytical approximation is valid. (Sharma N., et al, 2016)

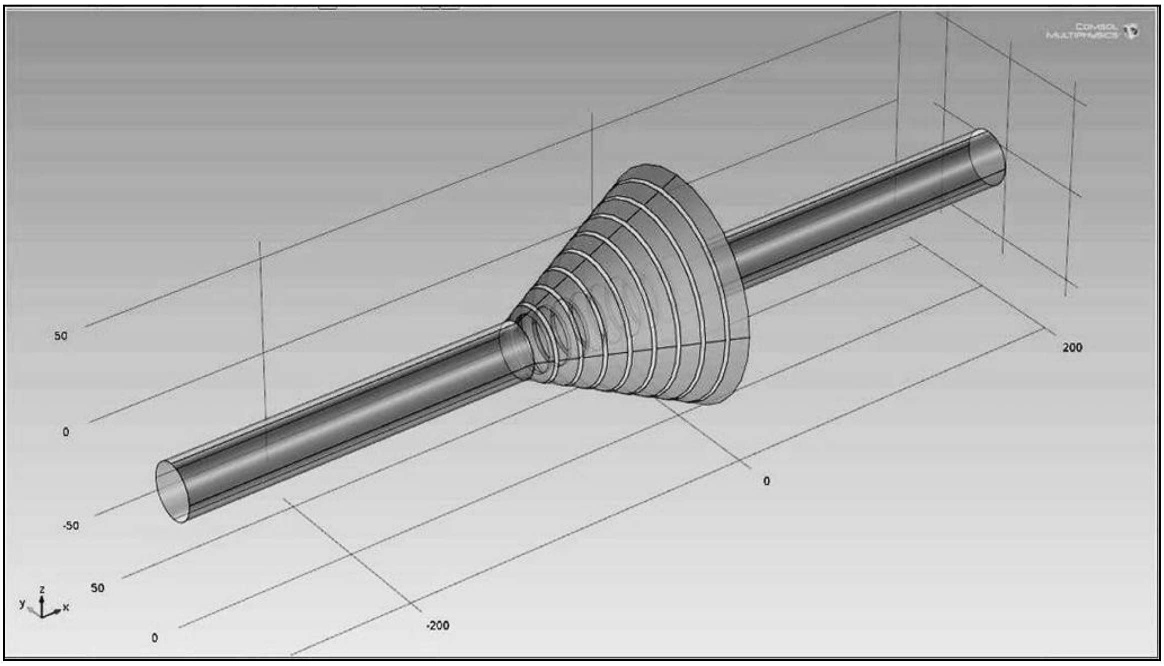

Figure 6.38: Simple Conical Chamber with internal structures

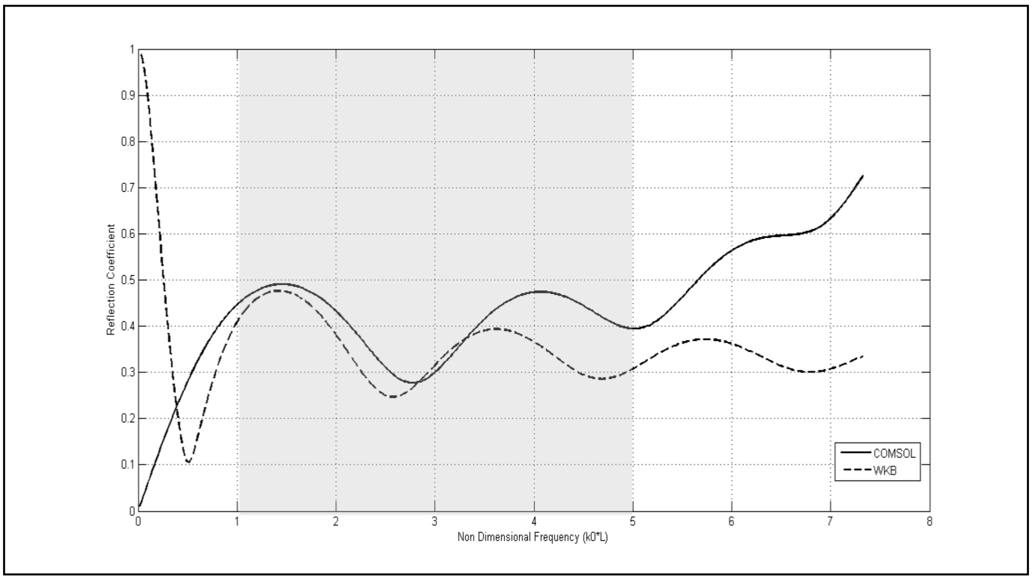

Figure 6.39: COMSOL versus WKB comparison plot for reflection coefficient against dimensionless frequency

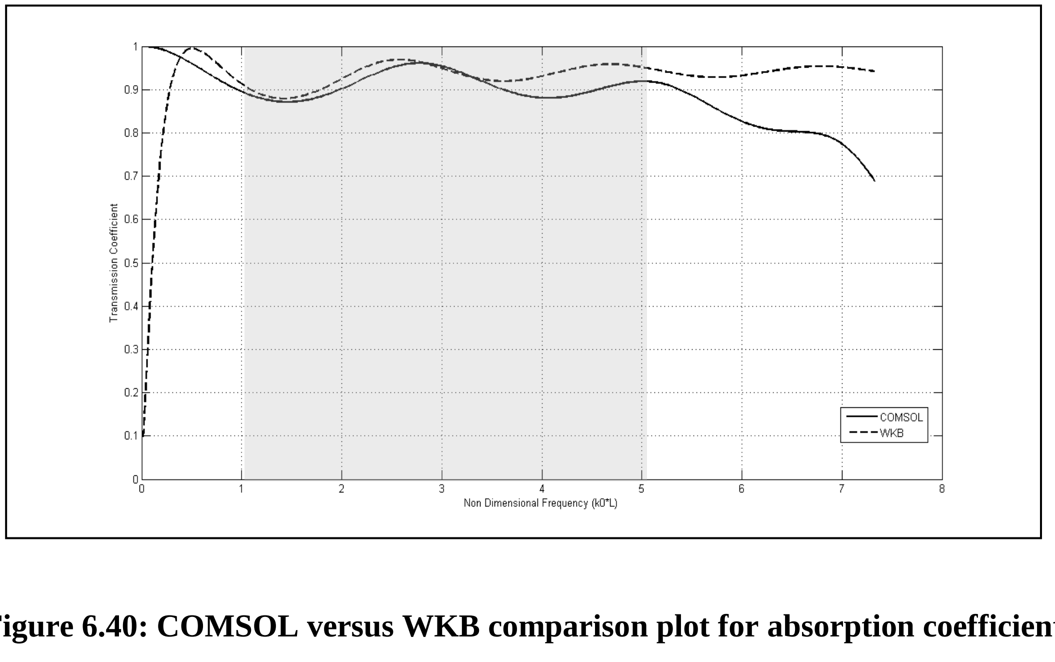

Figure 6.40: COMSOL versus WKB comparison plot for absorption coefficient against dimensionless frequency

6.2. Performance evaluation of muffler based on specific parametric variation

Having realized muffling configurations from a broad perspective in the previous section, a more detailed study has been discussed in this section. The main parameters that influence the performance of muffling and/or silencing sections have been individually varied. These essentially include; the shape of the flare based protrusion, the muffling chamber radius and the chamber length. The flare section encompasses a plurality of the ring and slit combination as also the presence of a lossy medium. The said parametrization can be depicted through the Figure 6.41. The impact of each of these parameters on the transmission and absorption capability has been detailed for general shape configurations.

6.2.1. Flare variation

The muffling section has been applied to study the impact of the lining in the presence and absence of the external flare. In addition, a comparison has also been made between the linear and the quadratic external flare shapes. The observations for a particular case have been compiled and presented for transmission loss and absorption coefficient. The dimensions are as follows:

$$\mathrm{r} = 0.05\ \mathrm{m};\ \mathrm{r_{min}} = \frac{\mathrm{r}}{2};\ \mathrm{R} = 2\mathrm{r};\ \mathrm{L} = 0.2\ \mathrm{m};\ \mathrm{h} = 0.002\ \mathrm{m};\ \mathrm{x} = 0.001\ \mathrm{m};\ \mathrm{N} = 50,$$

where $\mathrm{r_{min}}$ is minimum radius of the tube, Ris the maximum external radius, h is thickness of the rings and x is half distance between them.

From the comparative study, impact of the flare can be clearly understood. Figure 6.44 and Figure 6.42 show the transmission loss and absorption coefficient when the model is in transmission regime respectively. Figure 6.43 shows the reflection coefficient variation.

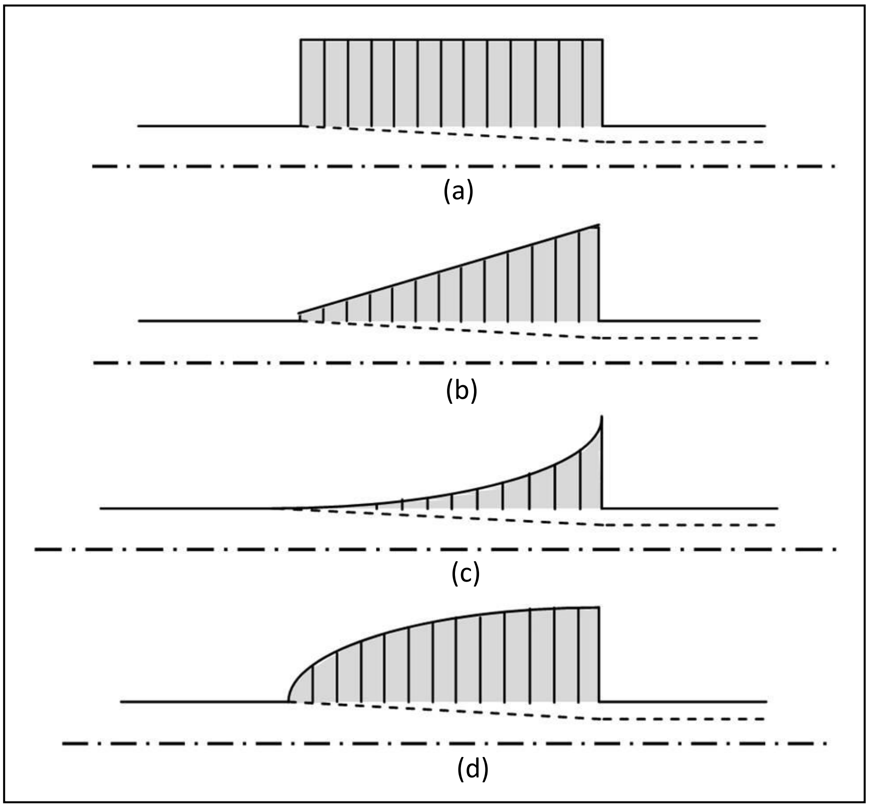

Figure 6.41: Depiction of parametrization of the flare shape with the presence of lossy medium (shaded sector) and ring and slit combination (a)No external flare with no or linear (dotted) internal flare; (b)Linear external flare with no or linear (dotted) internal flare; (c)Quadratic external flare with no or linear (dotted) internal flare; (d)Quadratic square root external flare with no or linear (dotted) internal flare

The absence and presence of flare section have been compared. It is evident that the presence of only an external flare has trivial impact quite similar to the case of no flare. The presence of an internal flare on the other hand, imparts a very strong boost to the coefficients. Of the three cases, where the internal flare is present, the case where only the internal flare is present, shows high values over a narrow range of frequency as compared to the case where the linear internal flare is balanced with the linear or quadratic external flare. This suggests the highly sensitive nature of the internal flare radius. Among the linear and the quadratic external flare, the quadratic is a steadier performer.

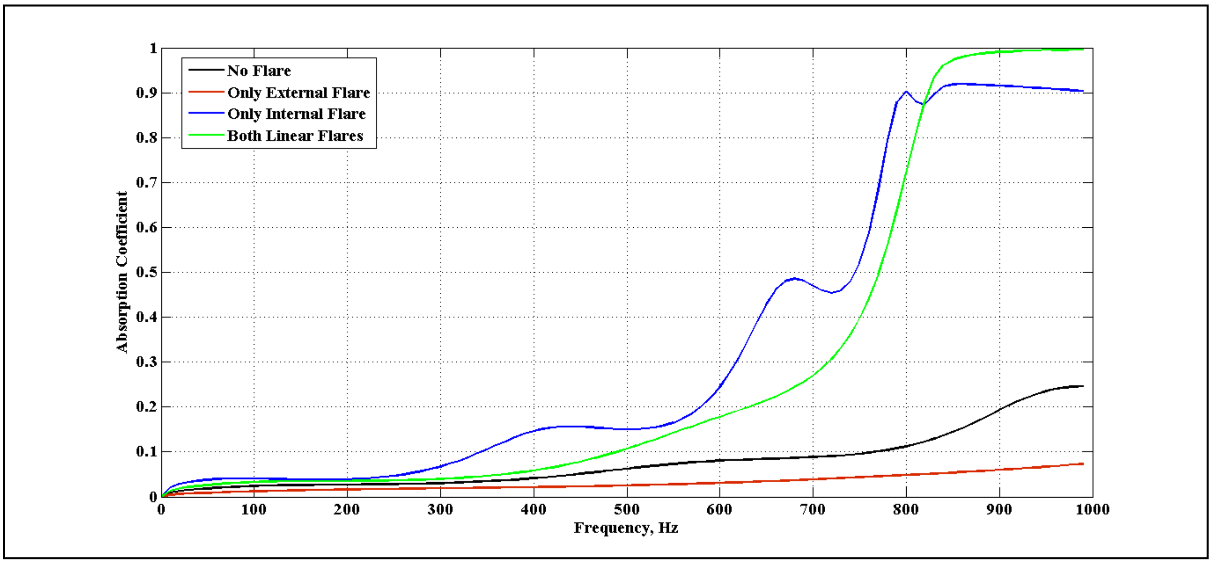

Figure 6.42:Absence of flare versus Linear Flare variation in terms of Absorption Coefficient against frequency (Hz)

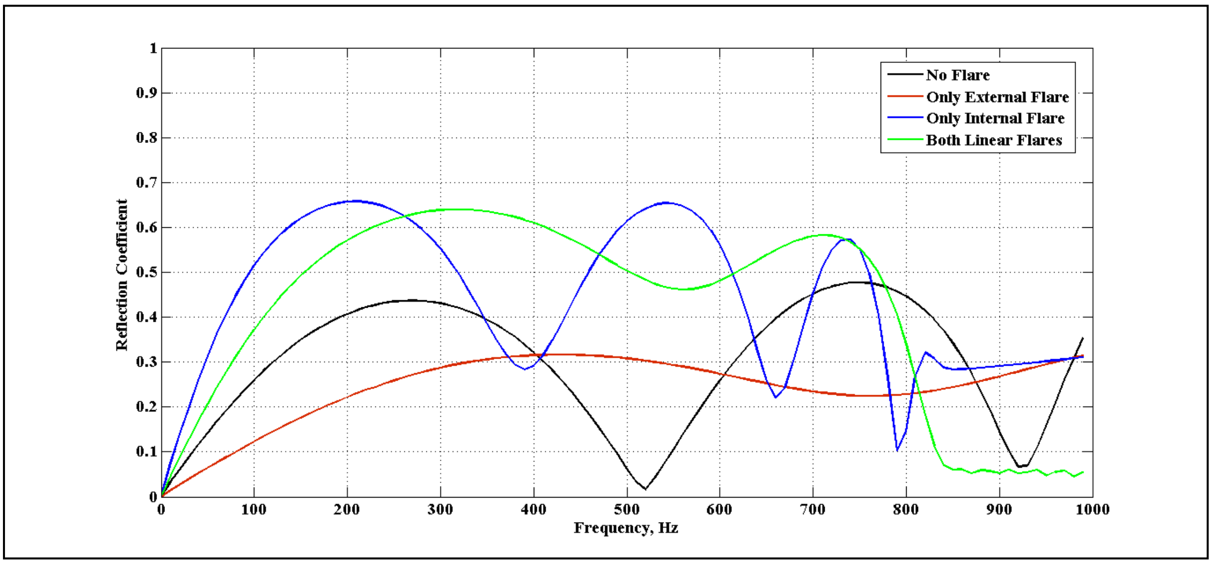

Figure 6.43: Absence of flare versus Linear Flare variation in terms of Reflection Coefficient against frequency (Hz)

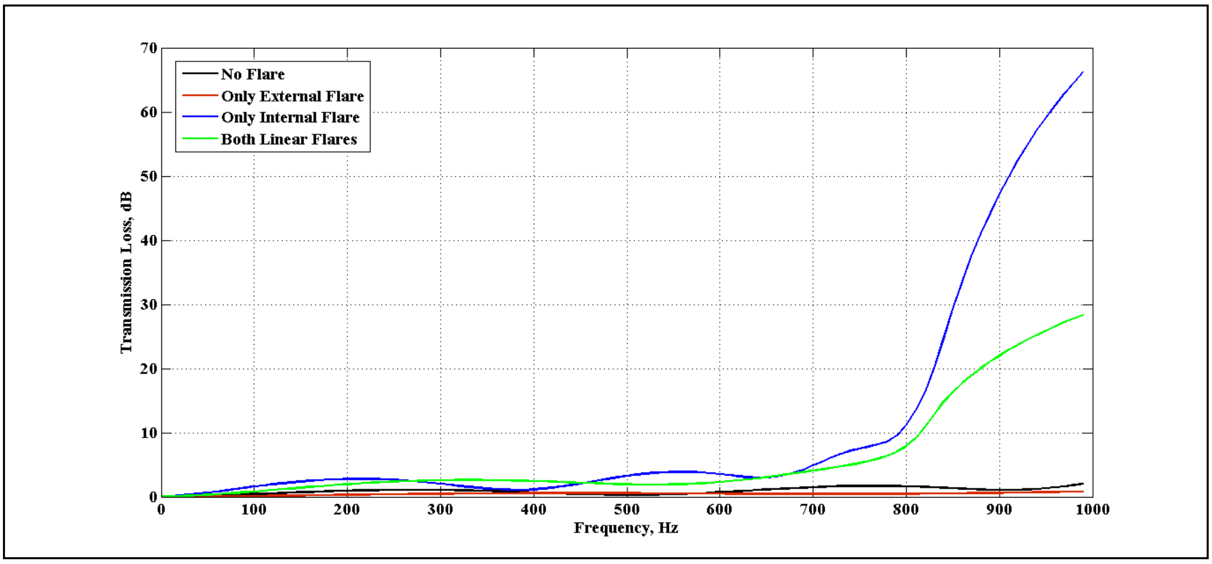

Figure 6.44: Absence of flare versus Linear Flare variation in terms of Transmission Loss (dB) against frequency (Hz)

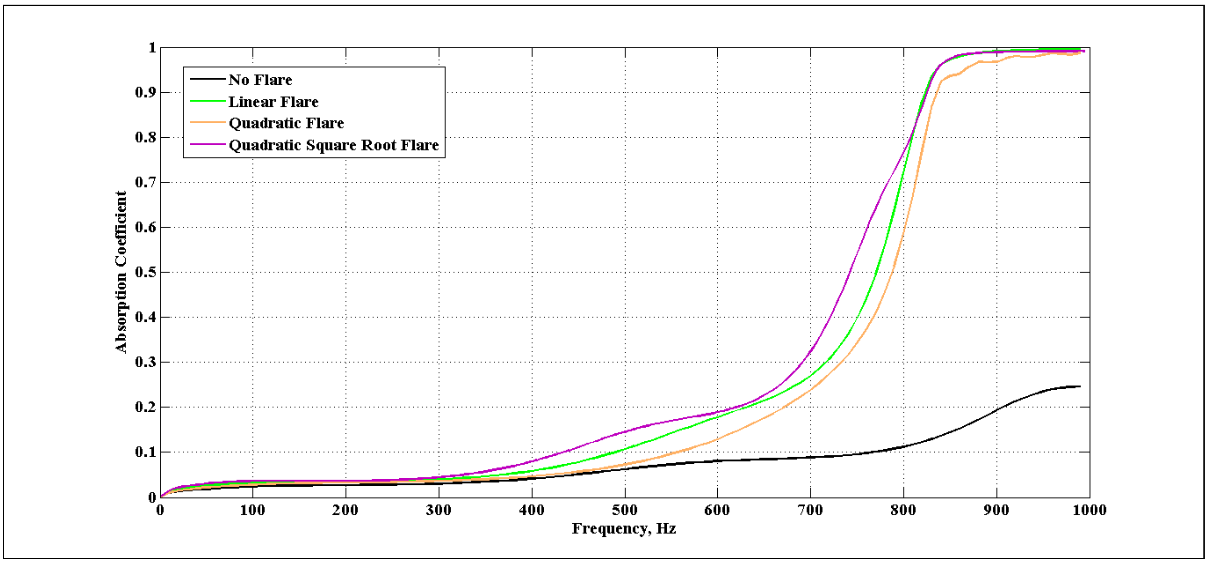

Figure 6.45: Absence of flare versus Power law function based Flare variation in terms of Absorption Coefficient against frequency (Hz)

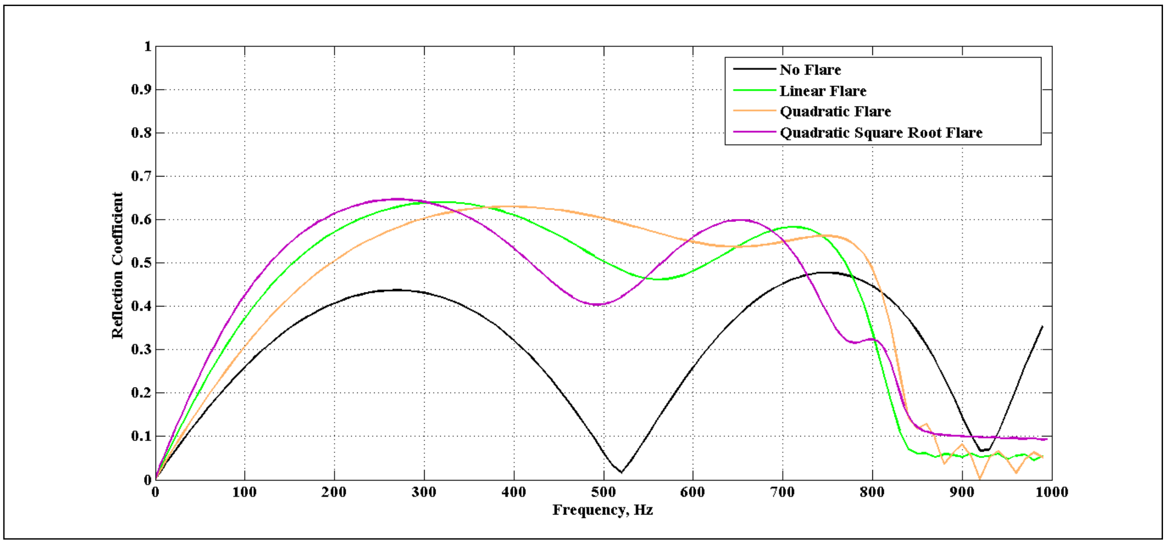

Figure 6.46: Absence of flare versus Power law function based Flare variation in terms of Reflection Coefficient against frequency (Hz)

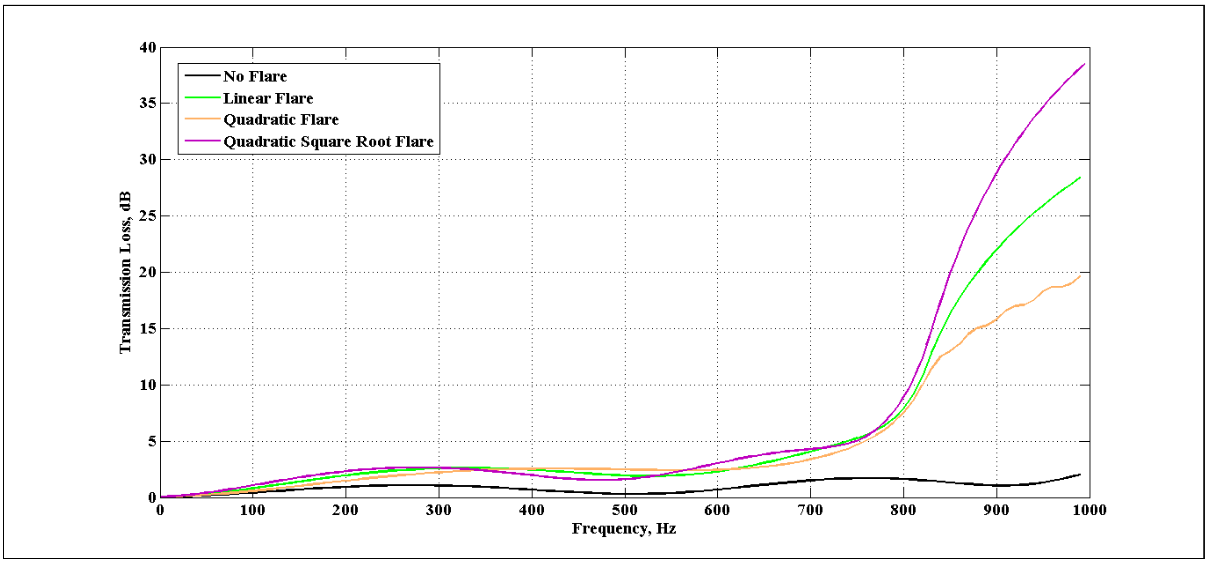

Figure 6.47: Absence of flare versus Power law function based Flare variation in terms of Transmission Loss (dB) against frequency (Hz)

However, of them all it is the quadratic square root shaped flare that can be considered as an outperformer. It not only has a comparable value to the only internal flare case but also is more stable due to its power-law function like profile. This supports the theory which states that a power-law variation has higher prominence over the approximated linear variation.

6.2.2. Chamber radius variation

From Figure 6.48 to Figure 6.56, a comparison between the systems while varying the radius of the chamber has been carried out for the performing configurations. Three radii, $r = 2\ cm, r = 5\ cm\ and\ r = 7\ cm$ have been plotted in combination for the performance parameters of the coefficients and the TL. A prominent general observation is that $r = 5\ cm$ is expectedly an intermediate performer in comparison to the others. The $2\ cm$ model shows its effect from much lower frequencies maintaining its effect all through; also suitable for broad frequency range applications. Being comparatively more stable in building a power loss across the section, $5\ cm$ radius tubes have been preferred for the study with losses. $7\ cm$ radius tubes show least effectiveness in all of the characteristic plots and have been preferably avoided.

6.2.2.1. Linear Flare

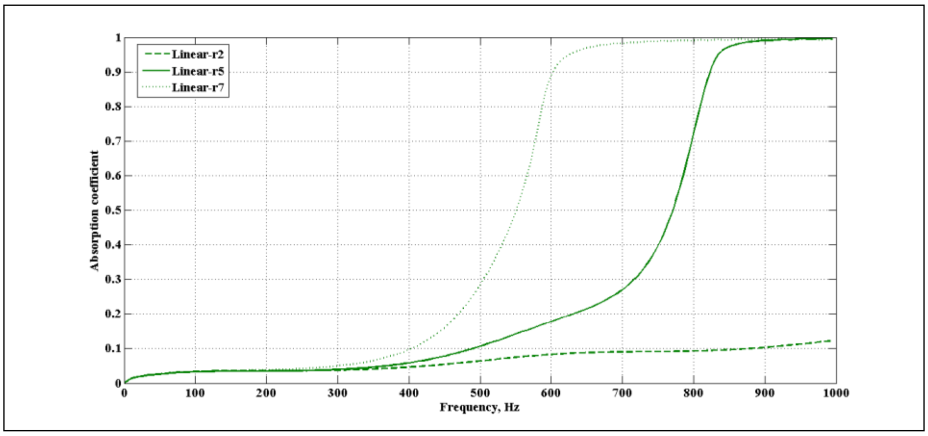

Figure 6.48: Chamber radius variations for linear flare in terms of Absorption Coefficient against frequency (Hz)

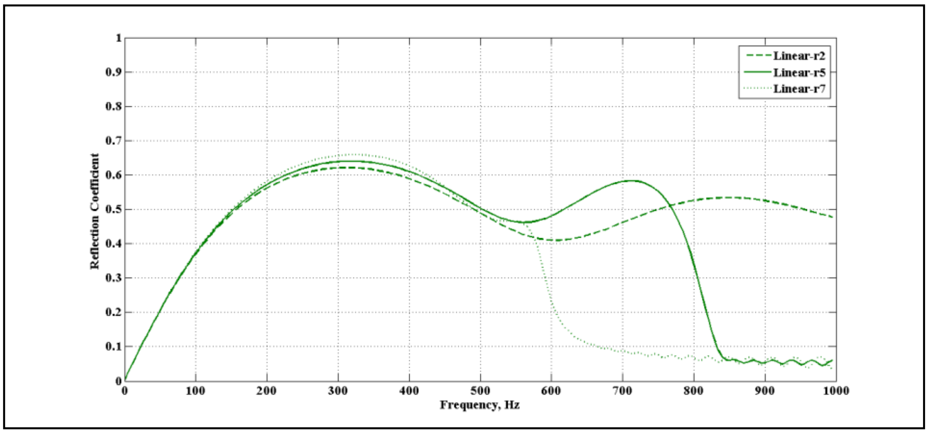

Figure 6.49: Chamber radius variations for linear flare in terms of Reflection Coefficient against frequency (Hz)

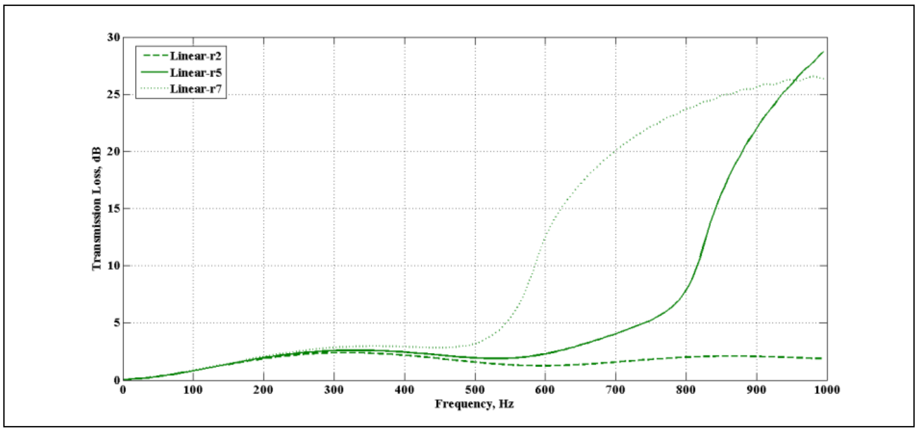

Figure 6.50: Chamber radius variations for linear flare in terms of Transmission loss (dB) against frequency (Hz)

6.2.2.2. Quadratic Flare

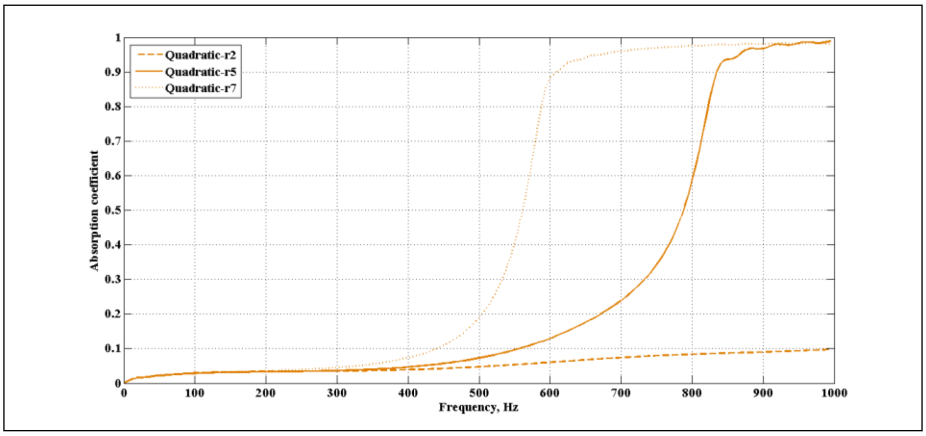

Figure 6.51: Chamber radius variations for quadratic flare in terms of Absorption Coefficient against frequency (Hz)

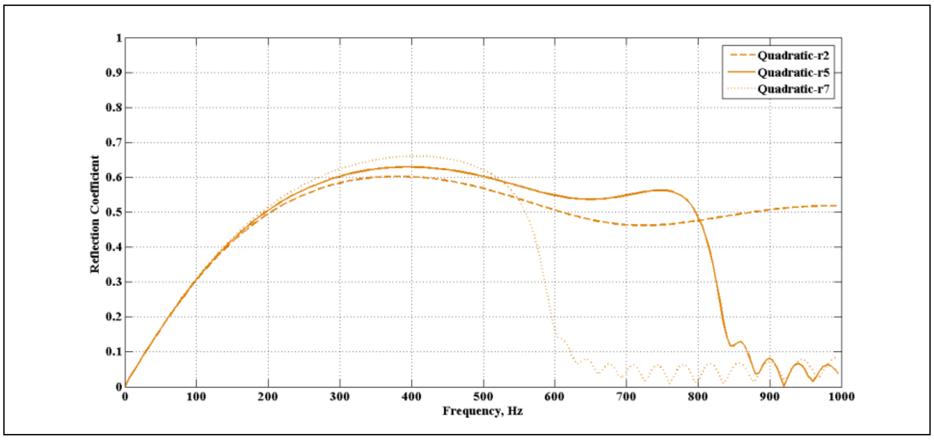

Figure 6.52: Chamber radius variations for quadratic flare in terms of Reflection Coefficient against frequency (Hz)

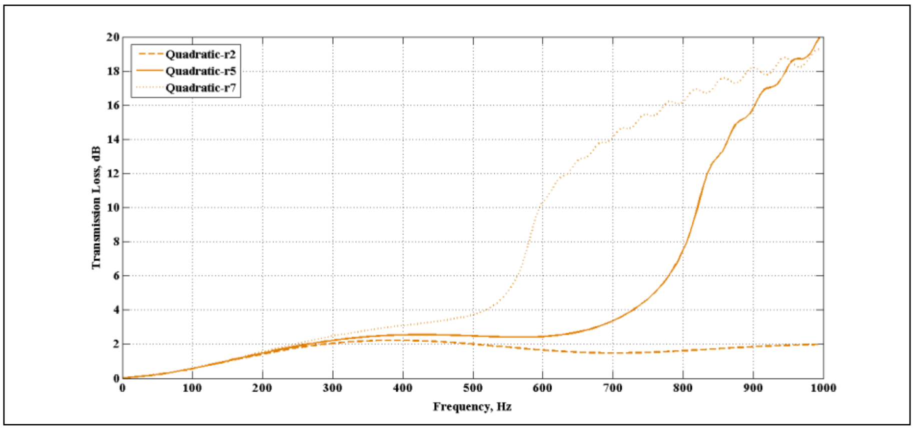

Figure 6.53: Chamber radius variations for quadratic flare in terms of Transmission Loss (dB) against frequency (Hz)

6.2.2.3. Quadratic Square Root Flare

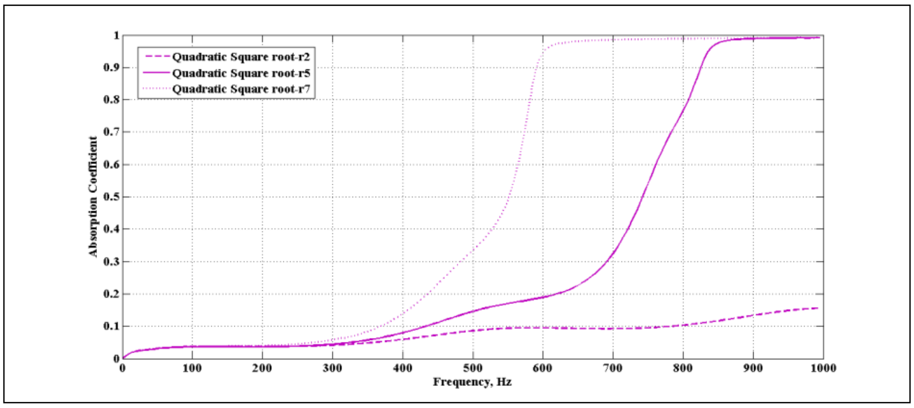

Figure 6.54: Chamber radius variations for quadratic square root flare in terms of Absorption Coefficient against frequency (Hz)

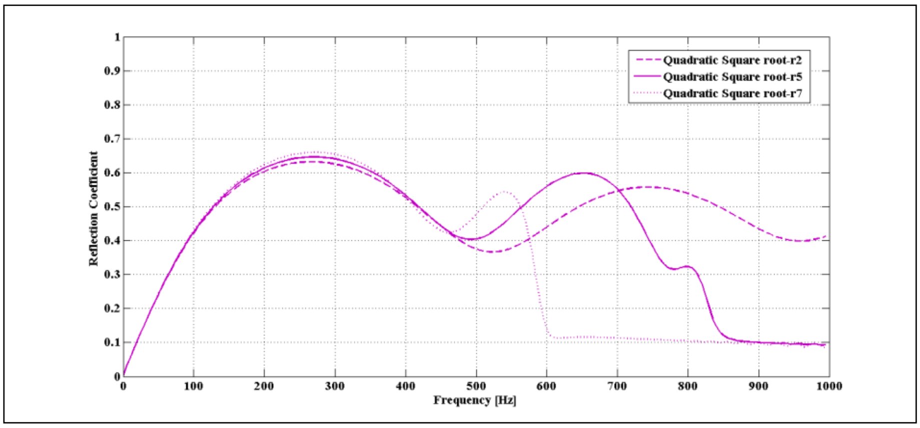

Figure 6.55: Chamber radius variations for quadratic square root flare in terms of Reflection Coefficient against frequency (Hz)

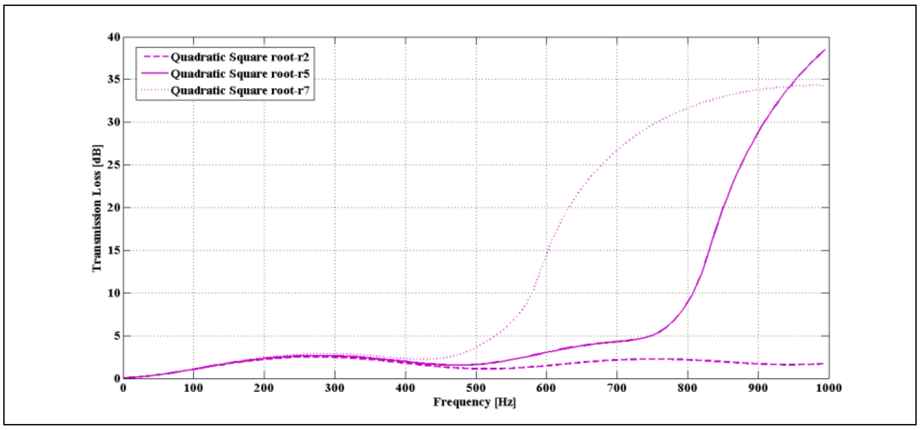

Figure 6.56: Chamber radius variations for quadratic square root flare in terms of Transmission Loss (dB) against frequency (Hz)

6.2.3. Chamber length variation

The length of the chamber has a direct connection to the number of ring and cavity units considered. The length of the chamber has been calculated as a function of the slit width and the cavity gap summing this for the number of such units.

Thus by varying the number the chamber length comparison has been carried out. For $N = 50$, the $l = 20\ cm$ or the short tube and by doubling the number to $N = 100$, the length doubles and acts as the long tube. These have been compared for the three configurations along the performance parameters (see Figures 6.57-6.59).

The curves do not show much variance in its trend. It can be observed that the shorter tubes generate smoother profiles in comparison to the longer ones. However, it can be said that the long quadratic square root flared tube is the best among the six variants. As evident, higher the number of sections better would be the muffling capacity of the system. Beyond the frequency of $800\ Hz$, the transmission loss curves begin to diverge and thus the absorption coefficient tends to a maximum.

6.3 Summary

Several parameters have been compared and analyzed for understanding the behaviour of absence, presence and variation of the major elements of the muffling chamber. While staying within the range of validity based on the plane wave approximation and solver based conditions, the observations as discussed through this section have a potential to guide in optimizing the design of mufflers and silencers.

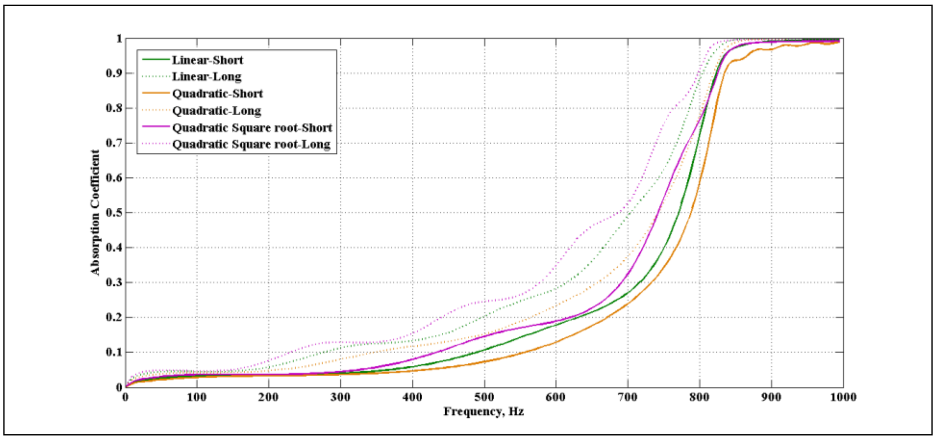

Figure 6.57: Chamber length variations in terms of Absorption coefficient against frequency (Hz)

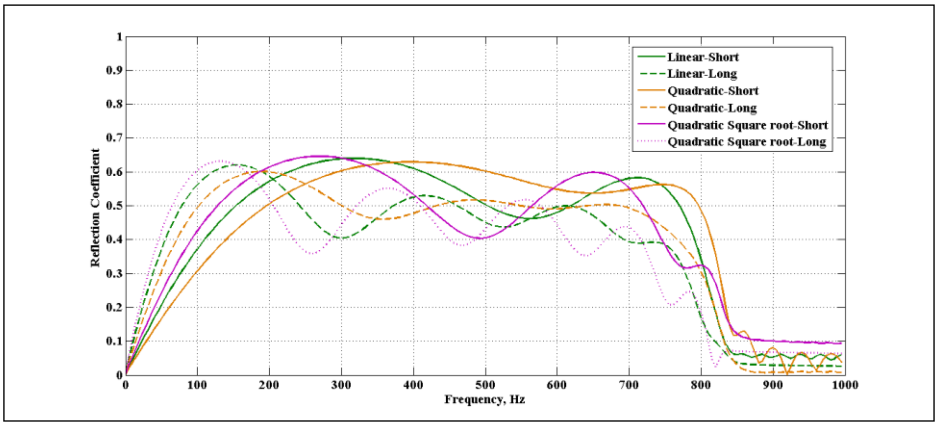

Figure 6.58: Chamber length variations in terms of Reflection coefficient against frequency (Hz)

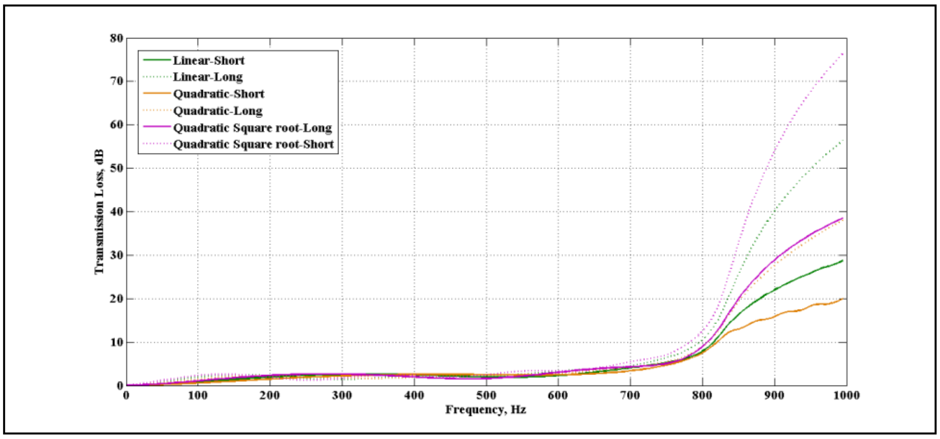

Figure 6.59: Chamber length variations in terms of Transmission Loss (dB) against frequency (Hz)

Chapter 7

Conclusion

The influence of acoustic metamaterial linings as an attenuation technique for designing the low frequency mufflers has been studied in this research project. A review of the available technologies and a theoretical understanding of the concepts associated with the modelling of the open termination muffler have been carried out to numerically analyze and validate the designs and their parametric variations. It has been concluded that the presence of a flaring extension onto the main muffling body comprising of graded wall admittance demonstrates better absorption and sound transmission loss characteristics over simple mufflers, when considering plane acoustic wave propagation through it in a lossy medium. The observations generated within this thesis could be helpful in modelling optimized low frequency muffler designs. The specific conclusions and relevant findings in support of these general conclusions have been thus presented.

A set of initial studies have been carried out for shape variations in the absence of internal losses within muffling sections. The shapes as defined included the simple circular, conical, quadratic and quadratic root based chambered mufflers. Characterizations for these basic systems were obtained in terms of the sound transmission loss, the reflection and absorption coefficients as also the FEM solver based results of acoustic pressure, local velocity and sound pressure levels. These characteristic plots have been generated against the TL peak and trough frequencies. It has been concluded that with reference to the circular chamber, the quadratic root chamber has comparable performance in the absence of a lossy medium. The power law function based models show superiority amongst the various designs considered.

To further this understanding, a study has been carried out to realise the impact of the presence on internal rigid structures such as the periodically arranged ring like slits. The medium entrapped in between the slits account for the graded wall admittance. The periodic structures are representative of metamaterial lining within the flaring section of the mufflers. It is in this regard that the analysis of these systems is governed by the principles based on the acoustic black hole effect. The comparative analysis between the semi analytical and FEM based approach show agreeable results within the range of applicability.

The next and final part of this project consists of the study involving the presence of visco-thermal energy exchanges within the medium. The acoustic plane wave propagation through systems bearing such internal medium has been analyzed for their low frequency responses. The muffler design performances have been derived in terms of flare profile variation and chamber dimension variations. It has been concluded that the linear flare is most effective in very low frequency region while the quadratic root based variations have steady overall performance. A chamber radius of dimensions of about a quarter of its length is considered stable while longer the chamber length, incorporating denser effective admittance is better in sound absorption and transmission loss.

Bibliography

Bängtsson, E., Noreland, D. and Berggren, M. (2003). Shape optimization of an acoustic horn. Computer Methods in Applied Mechanics and Engineering, 192(11-12), pp.1533-1571.

Beeching, C. (1965). Free flow acoustic silencer constructed of resilient material. US 3187837 A.

Bender, C. and Orszag, S. (2013). Advanced mathematical methods for scientists and engineers. 1st ed. New York: Springer.

Bentleypublishers.com. (2005). *VW - Volkswagen Technical Service Training: Noise, Vibration, Harshness - Bentley Publishers - Repair Manuals and Automotive Books*. [online] Available at: http://www.bentleypublishers.com/volkswagen/technical-training/vw-noise-vibration-harshness-ssp.html.

Beranek, L. (1971). Noise and vibration control. Edited by Leo L. Beranek. 1st ed. New York, etc.: McGraw-Hill Book Co.

Berglund, B., Hassmén, P. and Job, R. (1996). Sources and effects of low-frequency noise. *The Journal of the Acoustical Society of America*, 99(5), pp.2985-3002.

Bies, D. and Hansen, C. (2003). Engineering Noise Control. 1st ed. London: CRC Press.

El- Ouahabi, A., Krylov, V. and O'Boy, D. (2015). Investigation of the acoustic black hole termination for sound waves propagating in cylindrical waveguides. In: Inter-noise 2015.

Eriksson, G. (1981). Apparatus for damping noise from exhaust air outlets. US 4299305 A.

Ford Motor Co (1971). Vehicle muffler and particle separator. US 3559760 A.

Francis, C. and Barber, J. (2013). A framework for understanding noise impacts on wildlife: an urgent conservation priority. *Frontiers in Ecology and the Environment*, 11(6), pp.305-313.

Guasch, O., Arnela, M. and Sánchez-Martín, P. (2017). Transfer matrices to characterize linear and quadratic acoustic black holes in duct terminations. Journal of Sound and Vibration, 395, pp.65-79.

Honda Motor Co., Ltd. (2012). Resonant-type muffler. 20120261210 A1.

Jiménez, N., Huang, W., Romero-García, V., Pagneux, V. and Groby, J. (2016). Ultra-thin metamaterial for perfect and quasi-omnidirectional sound absorption. Applied Physics Letters, 109(12), p.121902.

Krylov, V. (2014). Acoustic black holes: recent developments in the theory and applications. IEEE Transactions on Ultrasonics, and Frequency Control, 61(8), pp.1296-1306.

Leclaire, P., Umnova, O., Dupont, T. and Panneton, R. (2015). Acoustical properties of air-saturated porous material with periodically distributed dead-end pores. The Journal of the Acoustical Society of America, 137(4), pp.1772-1782.

Leventhall, D. (2003). *A Review of Published Research on Low Frequency Noise and its Effects*. Defra Publications, London: Department for Environment, Food and Rural Affairs.

Mantyla, V. (2005). Sound muffling apparatus for air operated equipment. US 6902030 B2.

Middelberg, J. (2004). COMPUTATIONAL FLUID DYNAMICS ANALYSIS OF THE ACOUSTIC PERFORMANCE OF VARIOUS SIMPLE EXPANSION CHAMBER MUFFLERS. In: ACOUSTICS 2004. pp.123-127.

Mironov, M. and Pislyakov, V. (2002). One-dimensional acoustic waves in retarding structures with propagation velocity tending to zero. Acoustical Physics, 48(3), pp.347-352.

Morse, P., Ingard, K. and Beyer, R. (1969). Theoretical Acoustics. Journal of Applied Mechanics, 36(2), p.382.

Munjal, M and Eriksson L.J. (1989) Analysis of a hybrid noise control system for a duct. *Journal of Acoustical Society of America*, **86** (1), 832-834.

Munjal, M. and Thawani, P. (1997). Effect of protective layer on the performance of absorptive ducts. Noise Control Engineering Journal, 45(1), p.14.

Munjal, M. (2014). Acoustics of Ducts and Mufflers, 2nd Edition. 1st ed. John Wiley & Sons.

NASA/TM—2011-216995 (2011). Acoustic Absorption in Porous Materials. Glenn Research Center, Cleveland, Ohio.

Oldfield, R. (2006). IMPROVED MEMBRANE ABSORBERS. Degree of Master of Science by Research. Research Institute for the Built and Human Environment, Acoustics Department, University of Salford, Salford, UK.

Panigrahi, S. and Munjal, M. (2005). Combination mufflers - Theory and parametric study. *Noise Control Engineering Journal*, 53(6), p.247.

Pawsey, C. (2016). *Strategies to Absorb Aircraft Engine Noise*. 1st ed. Aircraft Noise and Vibration.

Pierce, A. (1995). Acoustics. 1st ed. New York: Acoustical Society of America.

Reeves, A. (1897). *Exhaust Muffler for Engines*. US582485 A.

Schmidt, H. (1918). Muffler.US 1274943 A.

Schnell, F. (1931). Exhaust muffler. US 1811762 A.

Sharma N., Umnova O., Moorhouse A. T., Analysis of a low frequency muffler based on the acoustic black hole effect, *Proceedings of the Institute of Acoustics*, 38 (1), (2016)

Sheet Delivery (1887). Machines. US 361376 A.

Snyder, S. (n.d.). Active Noise Control Primer. 1st ed. Modern Acoustics and Signal Processing.

Spon, E. (2004). Noise Control in Industry. 1st ed. London: Sound Research Laboratories Ltd.

Sterling, R. (2001). Pneumatic hand tool exhaust muffler. US 6209678 B1.

Stonestreet, H. (1959). http://www.google.co.uk/patents/US2877860. 2877860 A.

Umnova, O. and Zajamsek, B. (2012). Omnidirectional graded index sound absorber. In: Nantes Conference.

Visnapuu, A. and Lay, S. (1978). Muffler for pneumatic drill. US 4079809 A.

Webster, A. G. (1919). “Acoustical impedance, and the theory of horns and of the phonograph,” Proc. Natl. Acad. Sci. U.S.A. https://doi.org/PNASA65, 275–282. https://doi.org/PNASA6, Crossref, CAS

Wolfe, J. (n.d.). Helmholtz Resonance. [online] Newt.phys.unsw.edu.au. Available at: http://newt.phys.unsw.edu.au/jw/Helmholtz.html.

Wright, M. (2005). Lecture notes on the mathematics of acoustics. 1st ed. London: Imperial College Press.

Yasuda, T., Wu, C., Nakagawa, N. and Nagamura, K. (2013). Studies on an automobile muffler with the acoustic characteristic of low-pass filter and Helmholtz resonator. Applied Acoustics, 74(1), pp.49-57.